

Optics I: lenses and apertures

CS 478, Winter 2012



Slides courtesy of:

Marc Levoy
Computer Science Department
Stanford University

Announcements

- ◆ Class email list:
 - cs478-win1112-all@lists.stanford.edu
 - If you are auditing the course, you'll need to email the staff to get added to the mailing list:
 - cs478-win1112-staff@lists.stanford.edu
- ◆ No class on Monday -- MLK, Jr. Day
- ◆ Watch for announcement about final project in email later this week

Outline

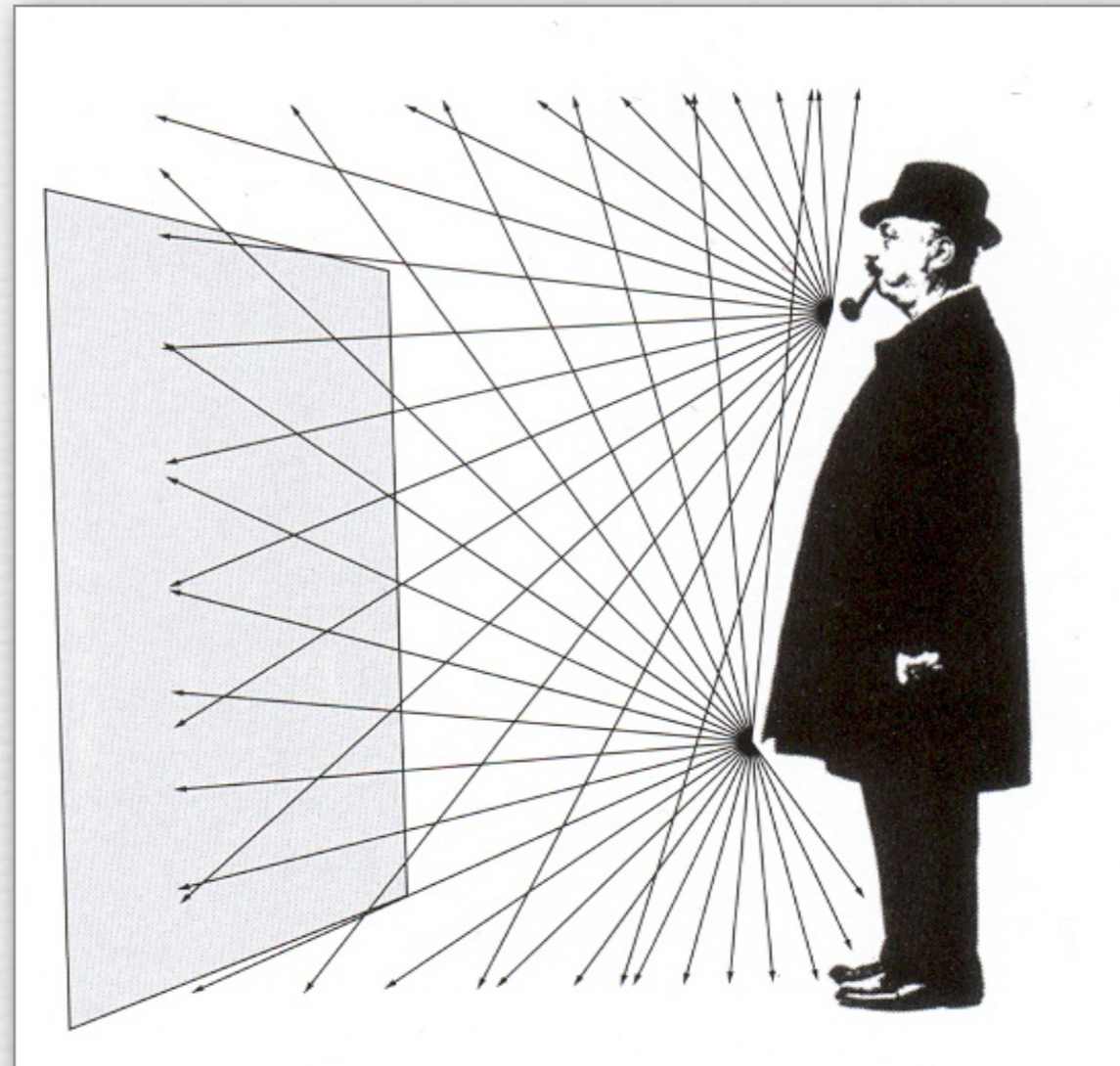
- ◆ why study lenses?
- ◆ thin lenses
 - graphical constructions, algebraic formulae
- ◆ thick lenses
 - lenses and perspective transformations
- ◆ depth of field
- ◆ aberrations & distortion
- ◆ vignetting, glare, and other lens artifacts
- ◆ diffraction and lens quality
- ◆ special lenses
 - telephoto, zoom

Single lens reflex camera (SLR)



Nikon F4
(film camera)

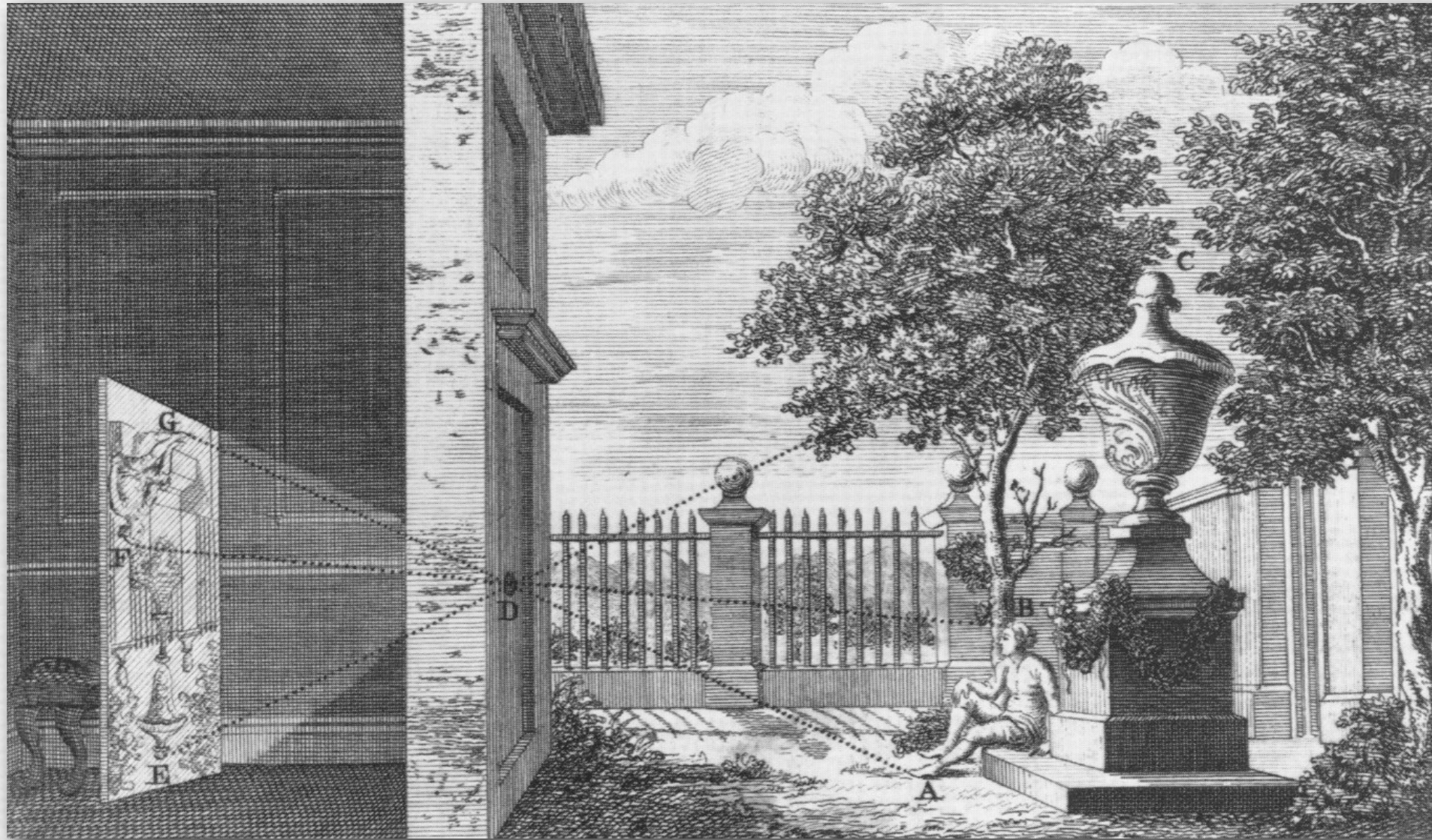
Why not use sensors without optics?



(London)

- ◆ each point on sensor would record the integral of light arriving from every point on subject
- ◆ all sensor points would record similar colors

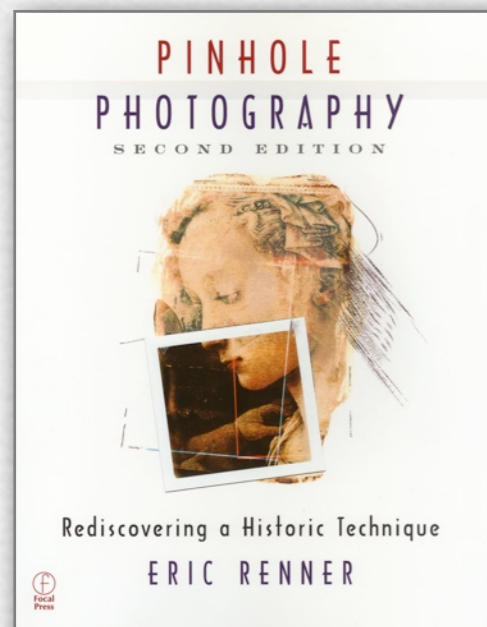
Pinhole camera (a.k.a. *camera obscura*)



- ◆ linear perspective with viewpoint at pinhole

Pinhole photography

- ◆ no distortion
 - straight lines remain straight
- ◆ infinite depth of field
 - everything is in focus



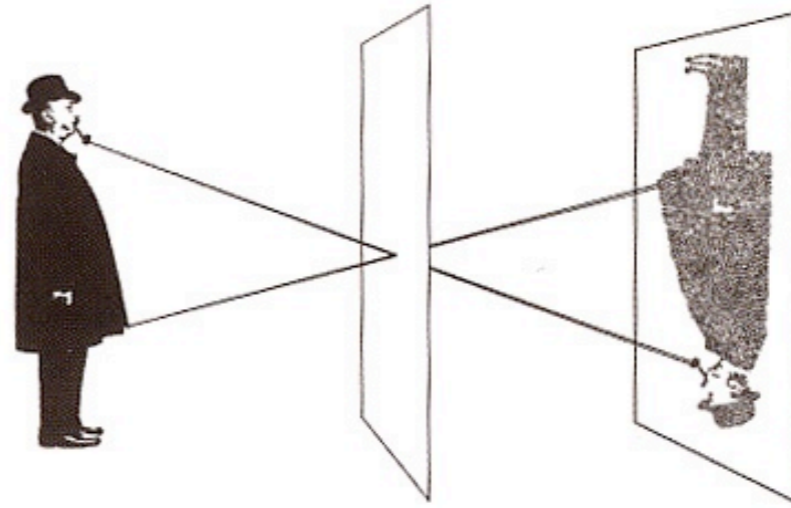
(Bami Adedoyin)



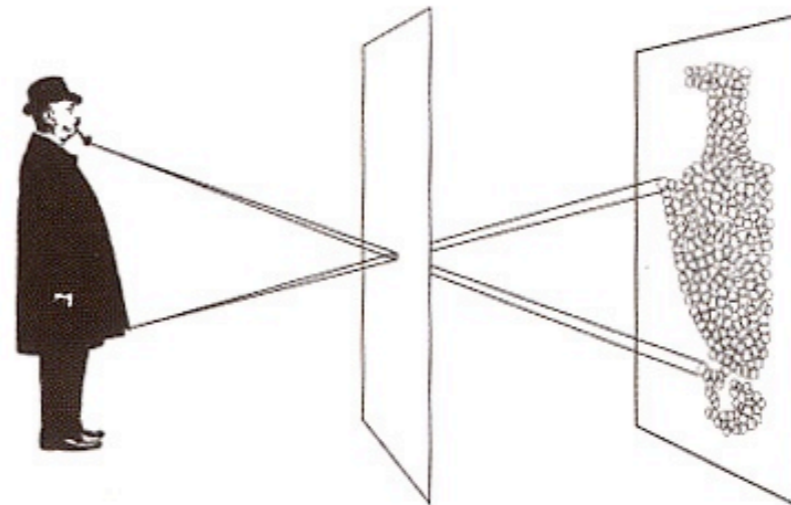
© Marc Levoy

Effect of pinhole size

Photograph made with small pinhole



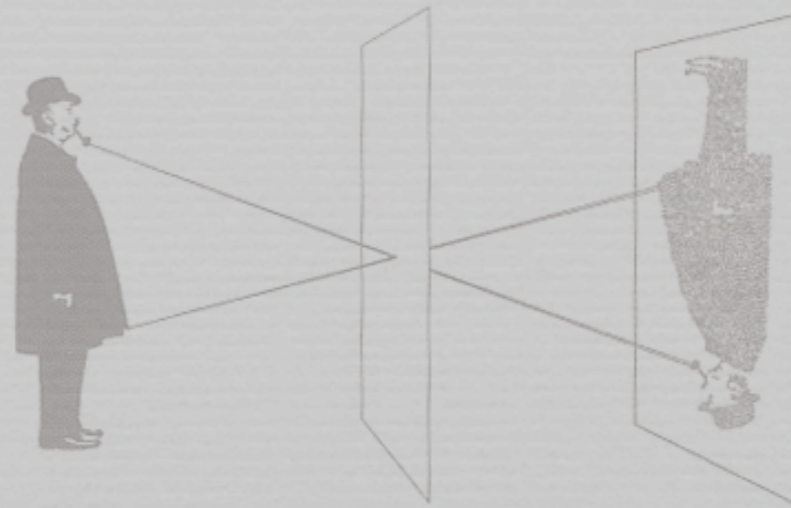
Photograph made with larger pinhole



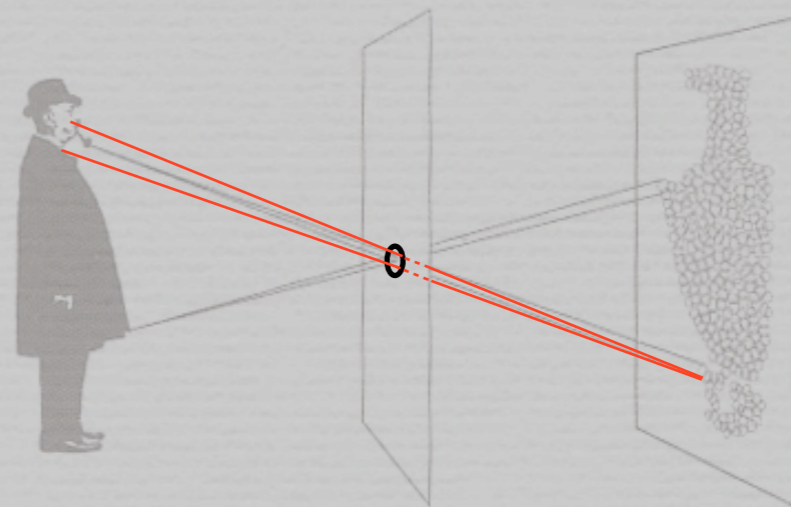
(London)

Effect of pinhole size

Photograph made with small pinhole



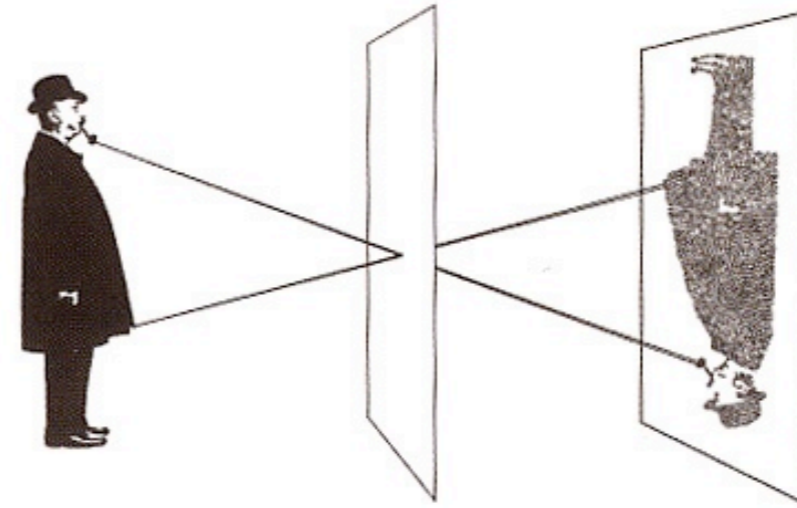
Photograph made with larger pinhole



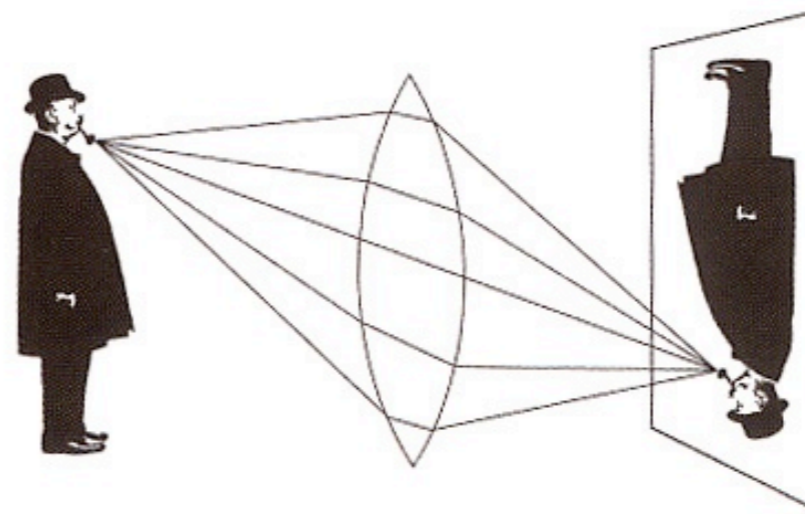
(London)

Replacing the pinhole with a lens

Photograph made with small pinhole

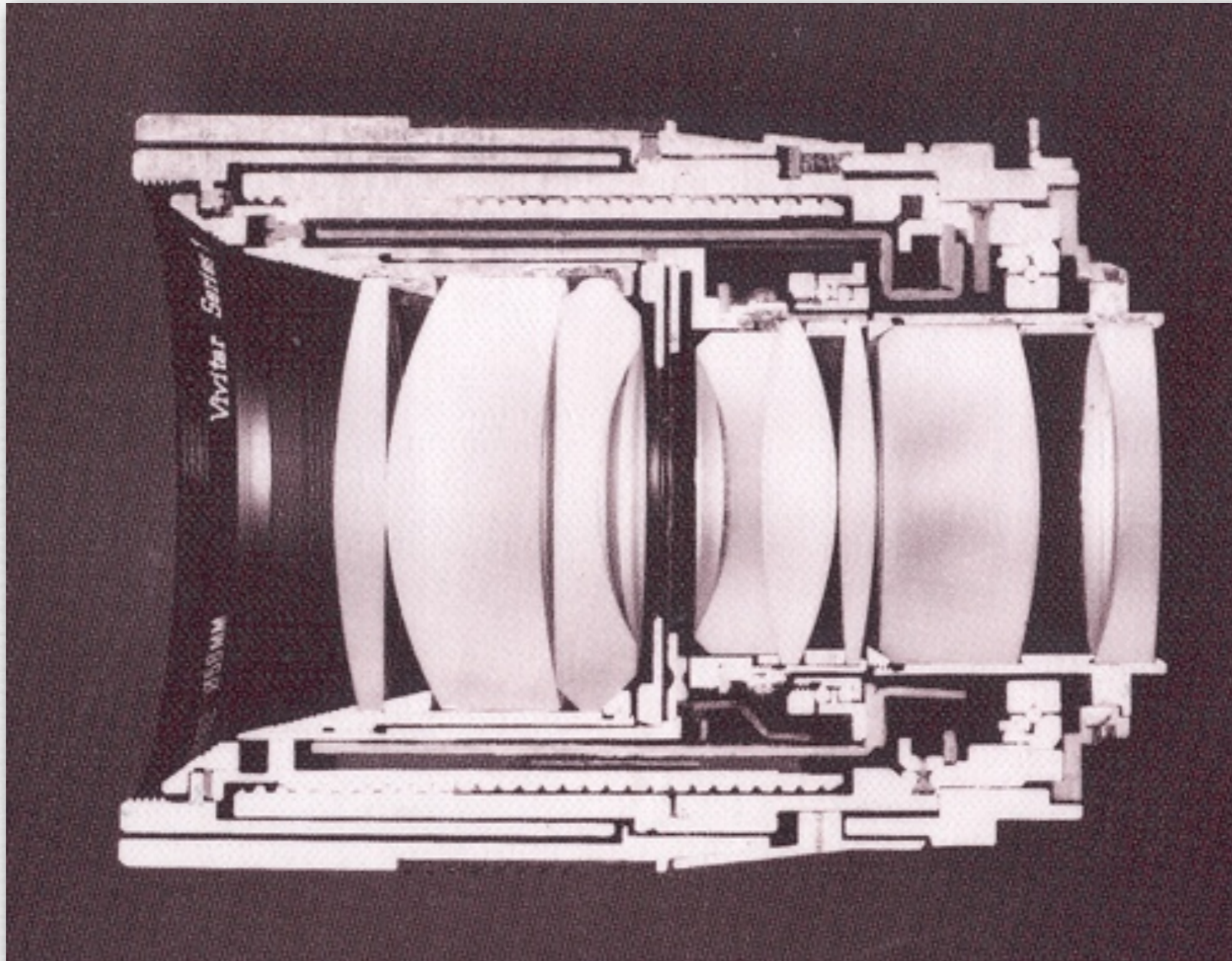


Photograph made with lens



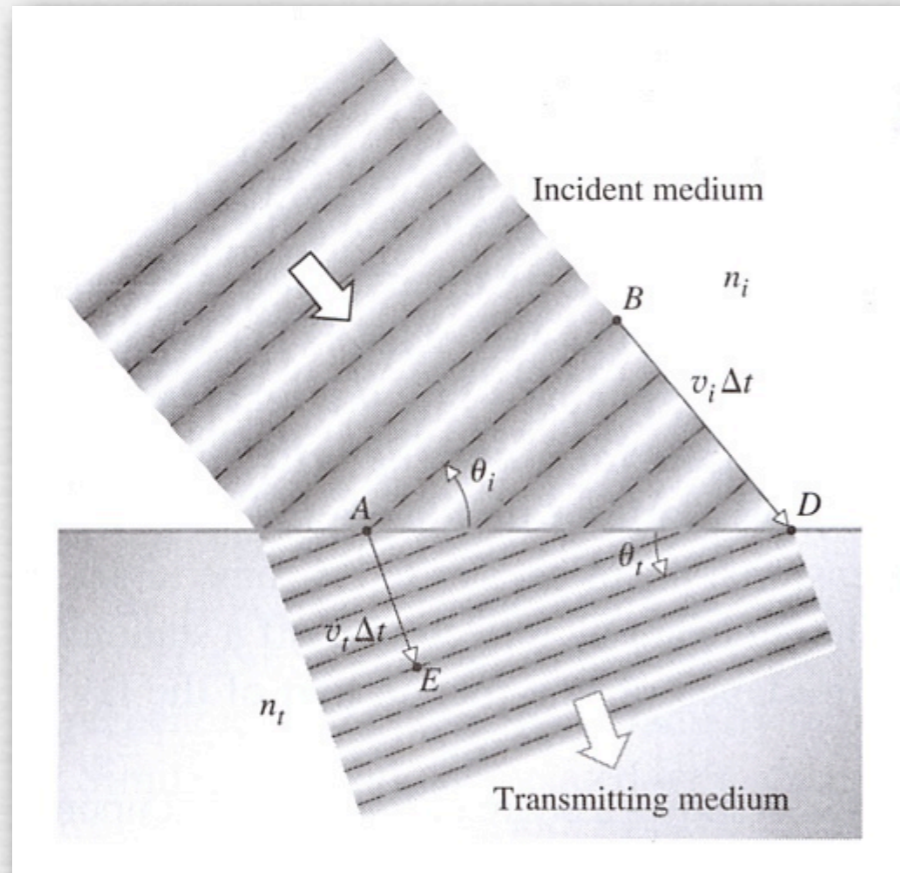
(London)

Cutaway view of a real lens

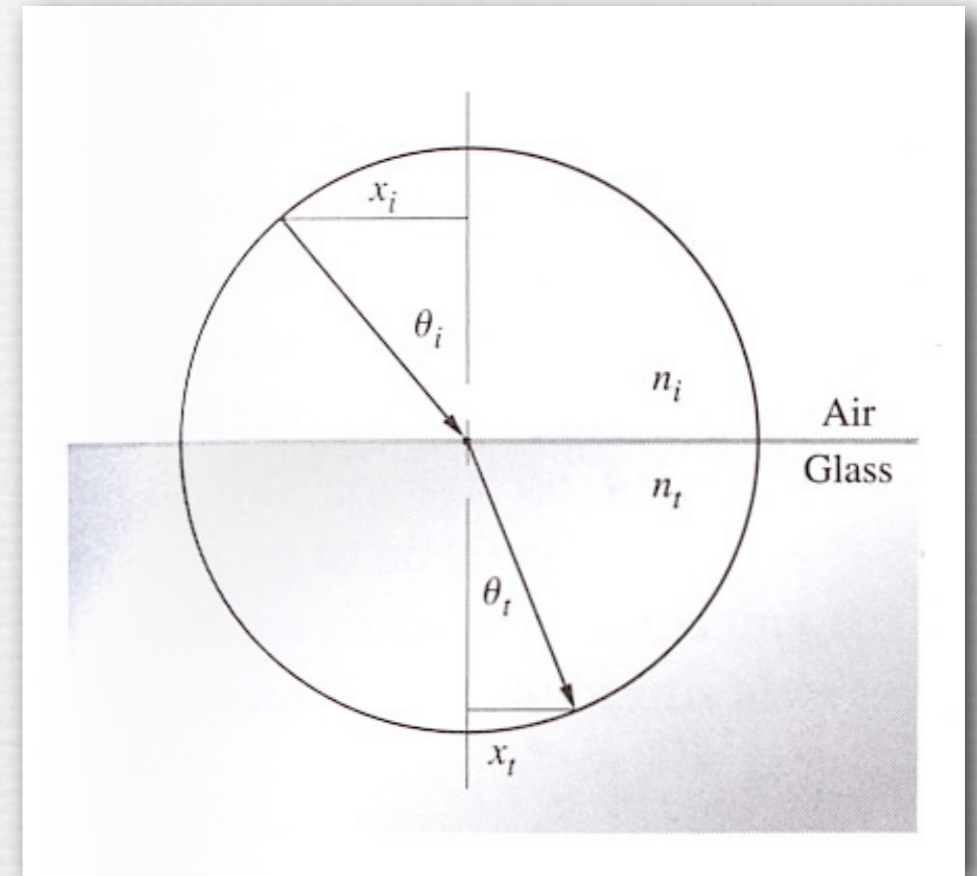


Vivitar Series 1 90mm f/2.5
Cover photo, Kingslake, *Optics in Photography*

Snell's law of refraction



(Hecht)



$$\frac{x_i}{x_t} = \frac{\sin \theta_i}{\sin \theta_t} = \frac{n_t}{n_i}$$

- ◆ as waves change speed at an interface, they also change direction
- ◆ index of refraction n_t is defined as

$\frac{\text{speed of light in a vacuum}}{\text{speed of light in medium } t}$

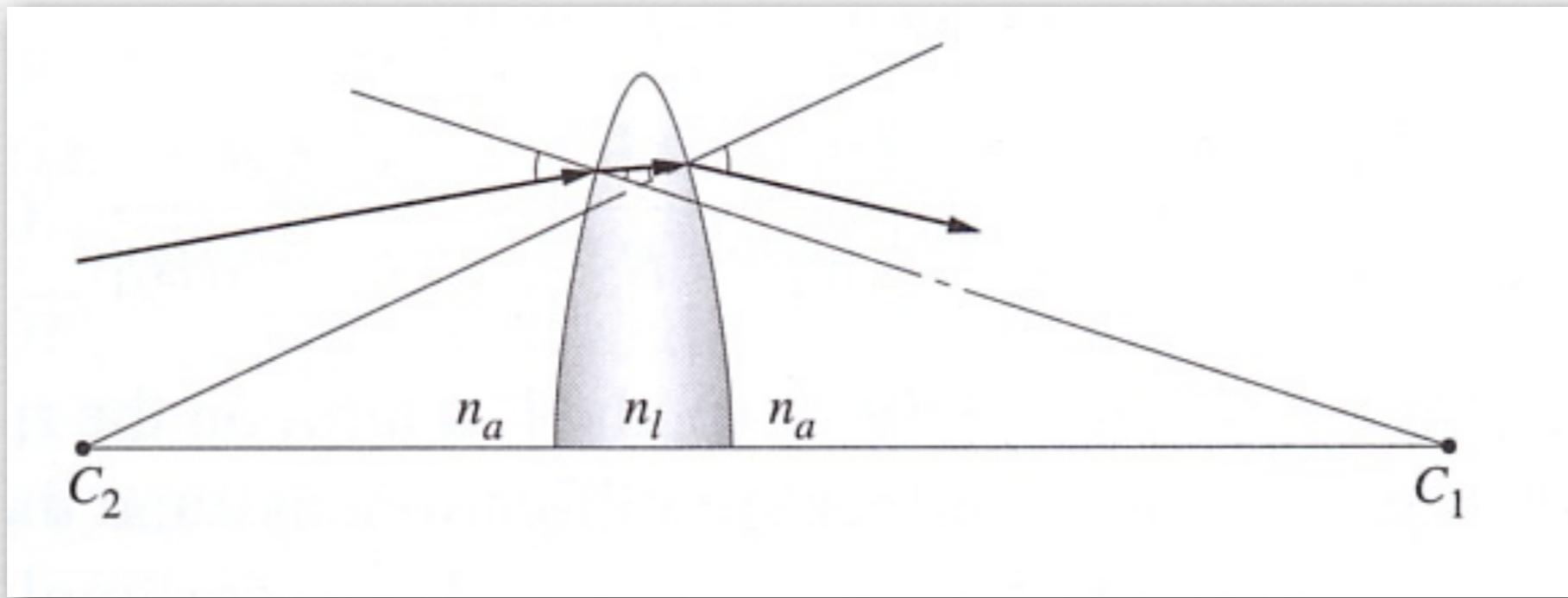
Typical refractive indices (n)

- ◆ air = 1.0
- ◆ water = 1.33
- ◆ glass = 1.5 - 1.8



mirage due to changes in the index of refraction of air with temperature

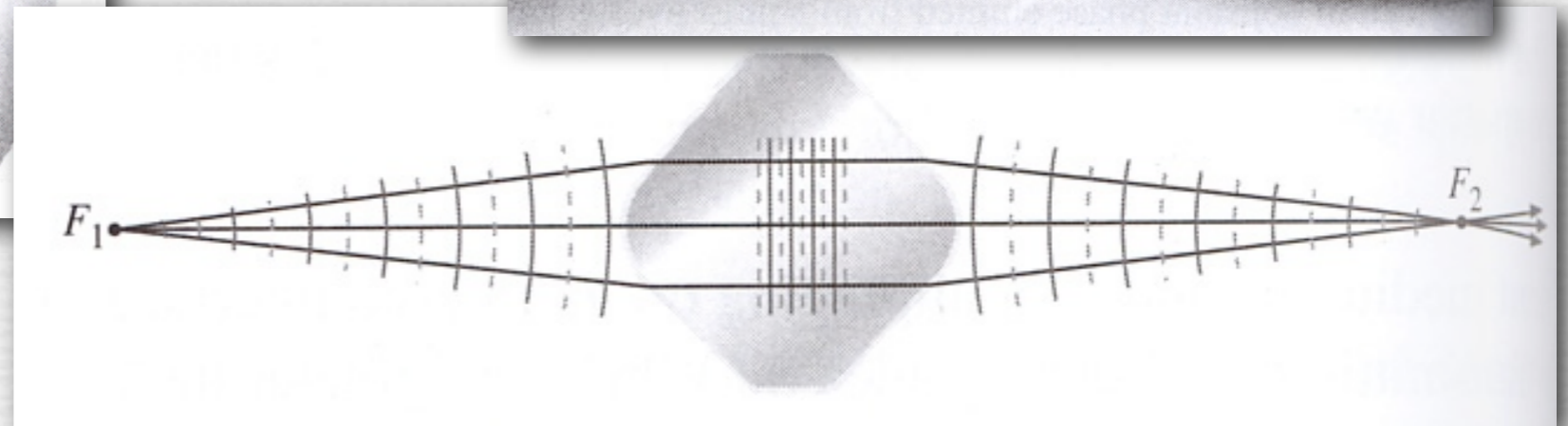
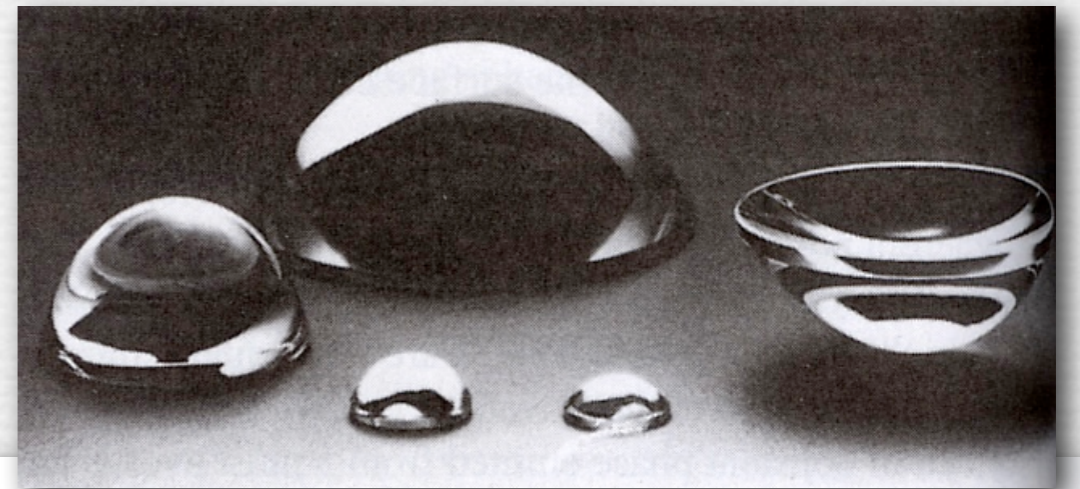
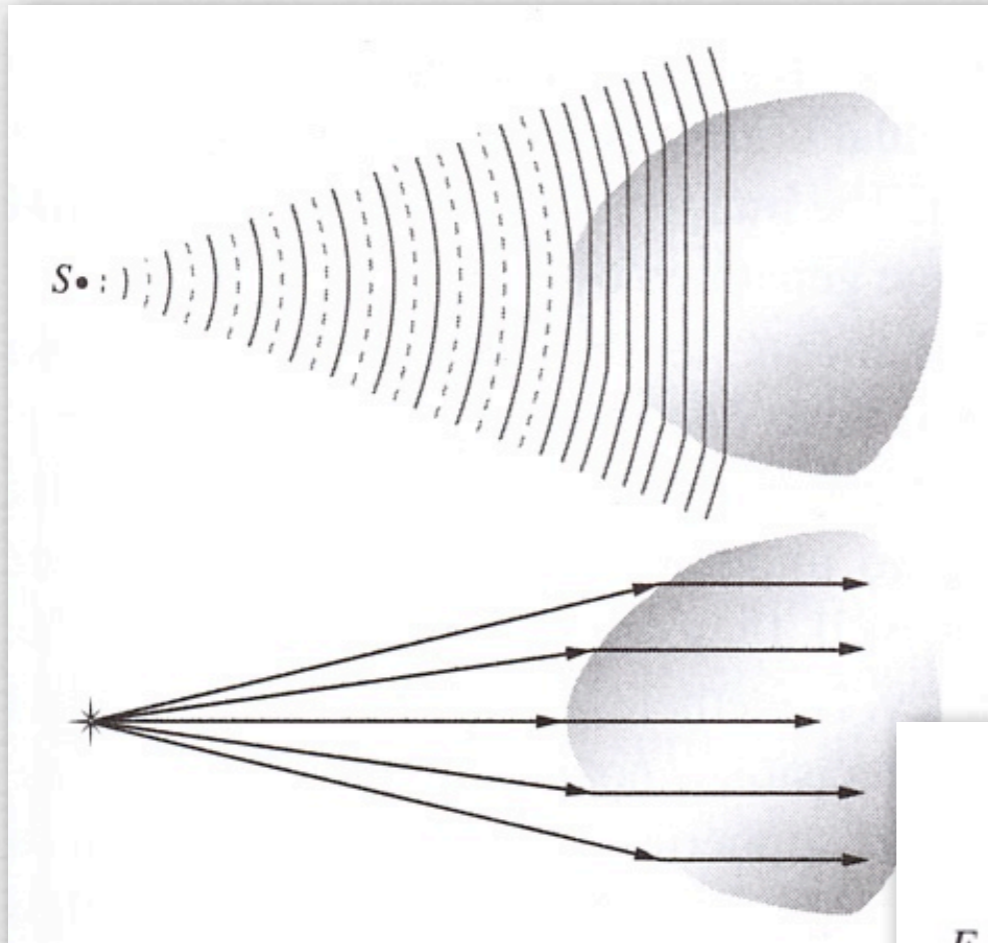
Refraction in glass lenses



(Hecht)

- ◆ when transiting from air to glass, light bends towards the normal
- ◆ when transiting from glass to air, light bends away from the normal
- ◆ light striking a surface perpendicularly does not bend

Q. What shape should an interface be to make parallel rays converge to a point?



(Hecht)

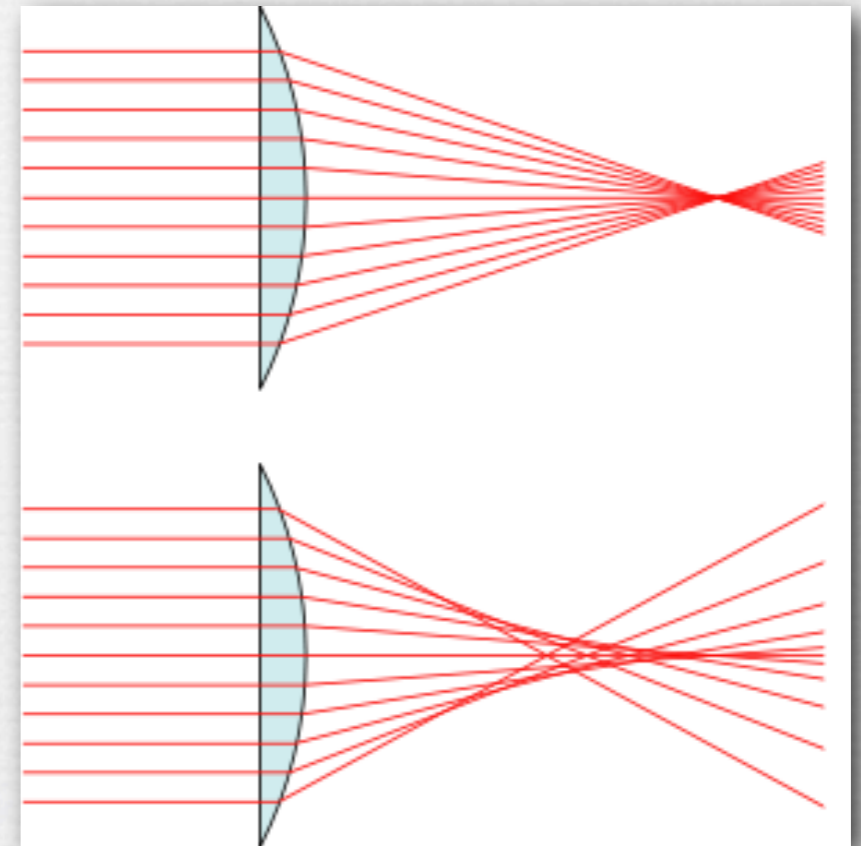
A. a hyperbola

◆ so lenses should be hyperbolic!

Spherical lenses



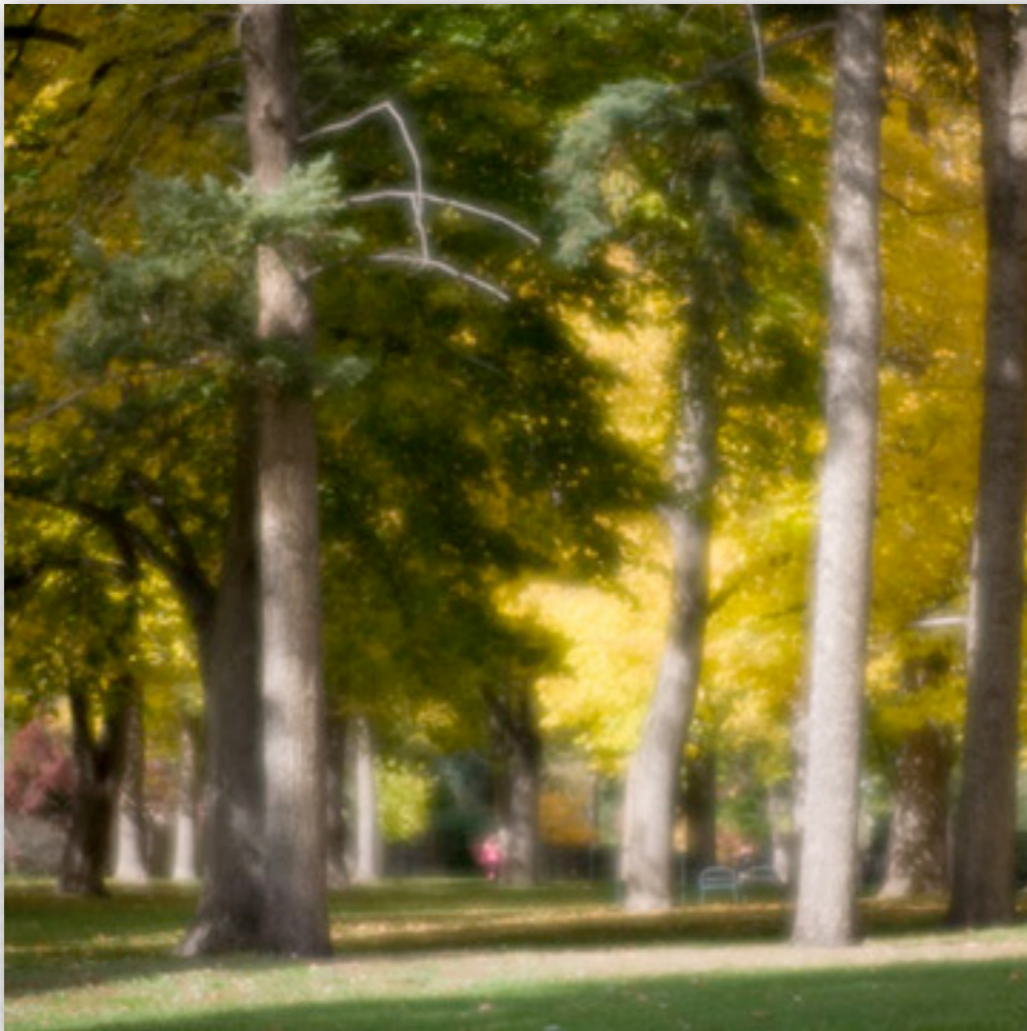
(Hecht)



(wikipedia)

- ◆ two roughly fitting curved surfaces ground together will eventually become spherical
- ◆ spheres don't bring parallel rays to a point
 - this is called *spherical aberration*
 - nearly axial rays (*paraxial rays*) behave best

Examples of spherical aberration



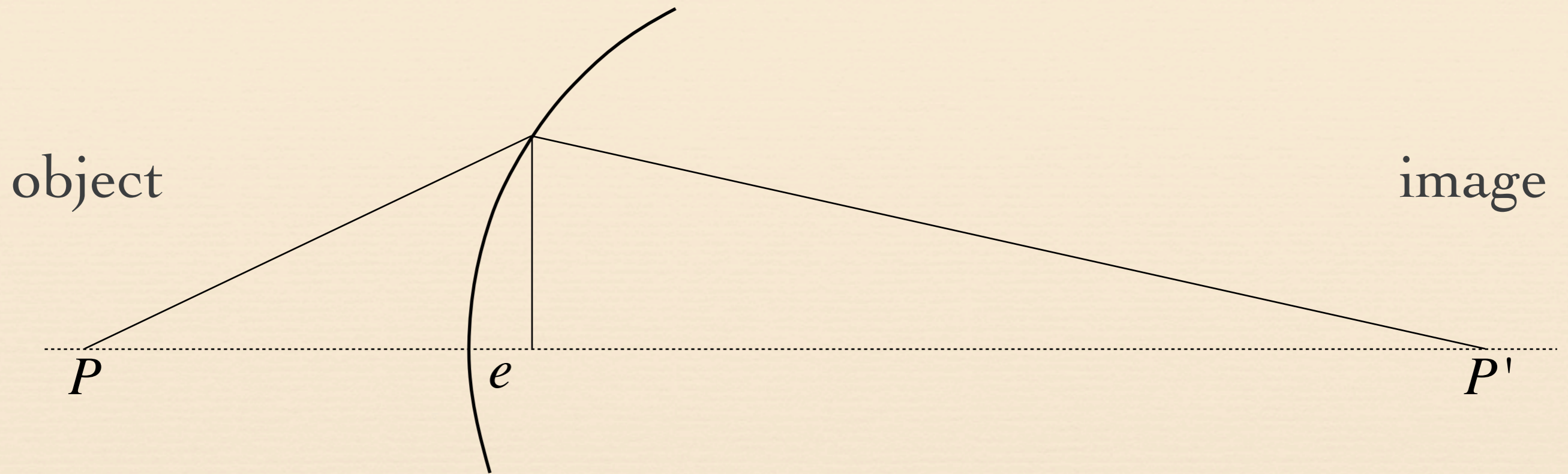
(gtmerideth)



(Canon)

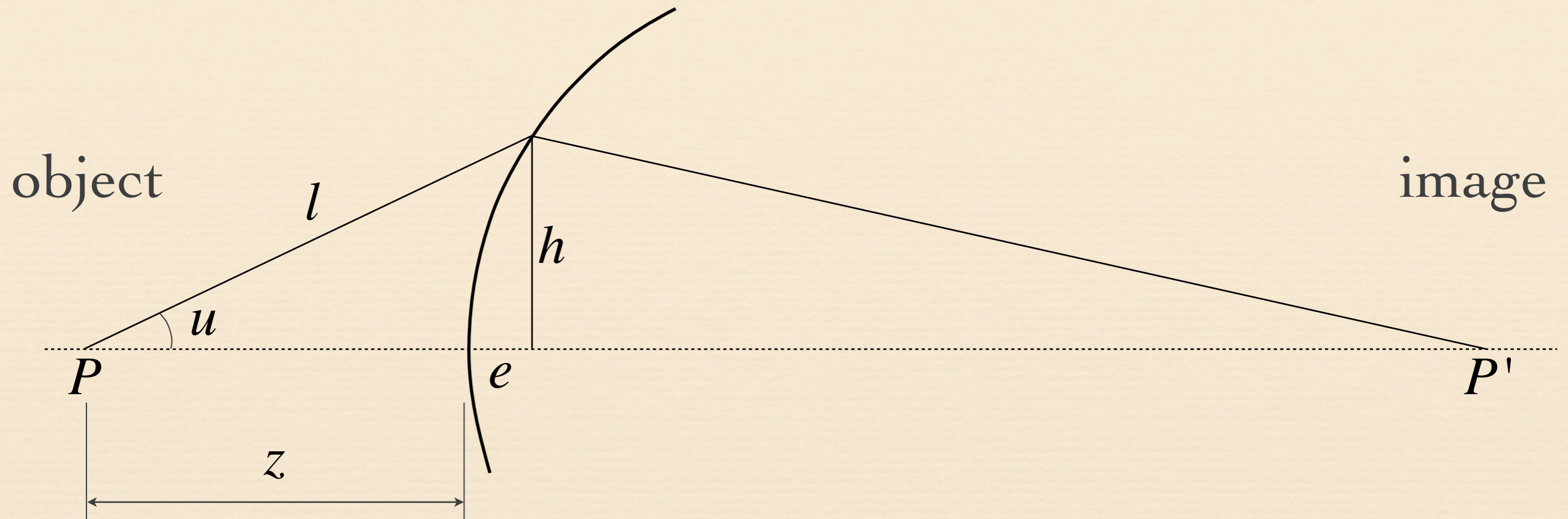


Paraxial approximation



◆ assume $e \approx 0$

Paraxial approximation



◆ assume $e \approx 0$

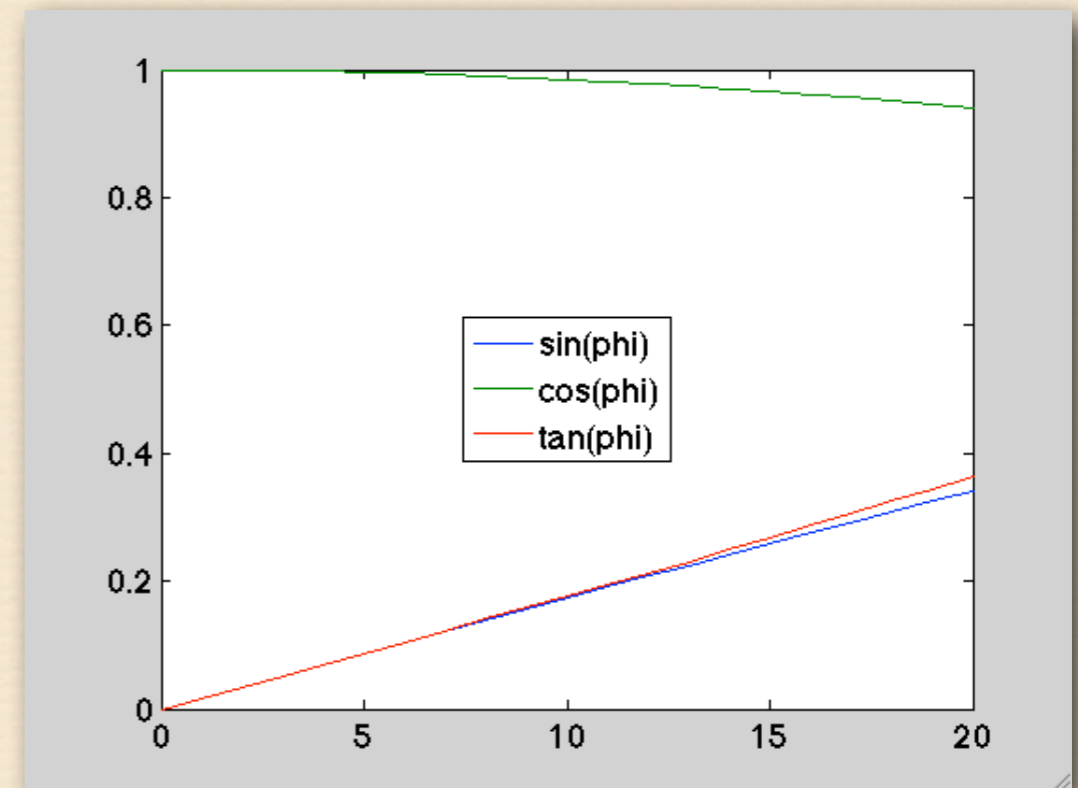
◆ assume $\sin u = h/l \approx u$ (for u in radians)

◆ assume $\cos u \approx z/l \approx 1$

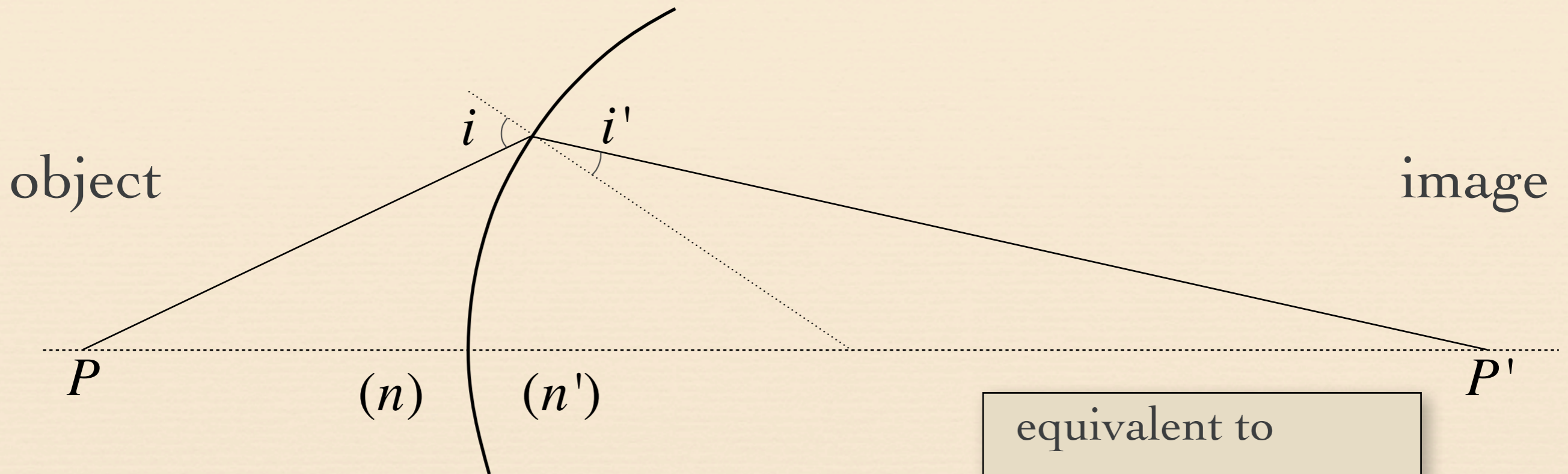
◆ assume $\tan u \approx \sin u \approx u$

The paraxial approximation is a.k.a. first-order optics

- ◆ assume first term of $\sin \phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \dots$
 - i.e. $\sin \phi \approx \phi$
- ◆ assume first term of $\cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \dots$
 - i.e. $\cos \phi \approx 1$
 - so $\tan \phi \approx \sin \phi \approx \phi$



Paraxial focusing



Snell's law:

$$n \sin i = n' \sin i'$$

paraxial approximation:

$$n i \approx n' i'$$

equivalent to

$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{n_t}{n_i}$$

with

$$n = n_i \text{ for air}$$

$$n' = n_t \text{ for glass}$$

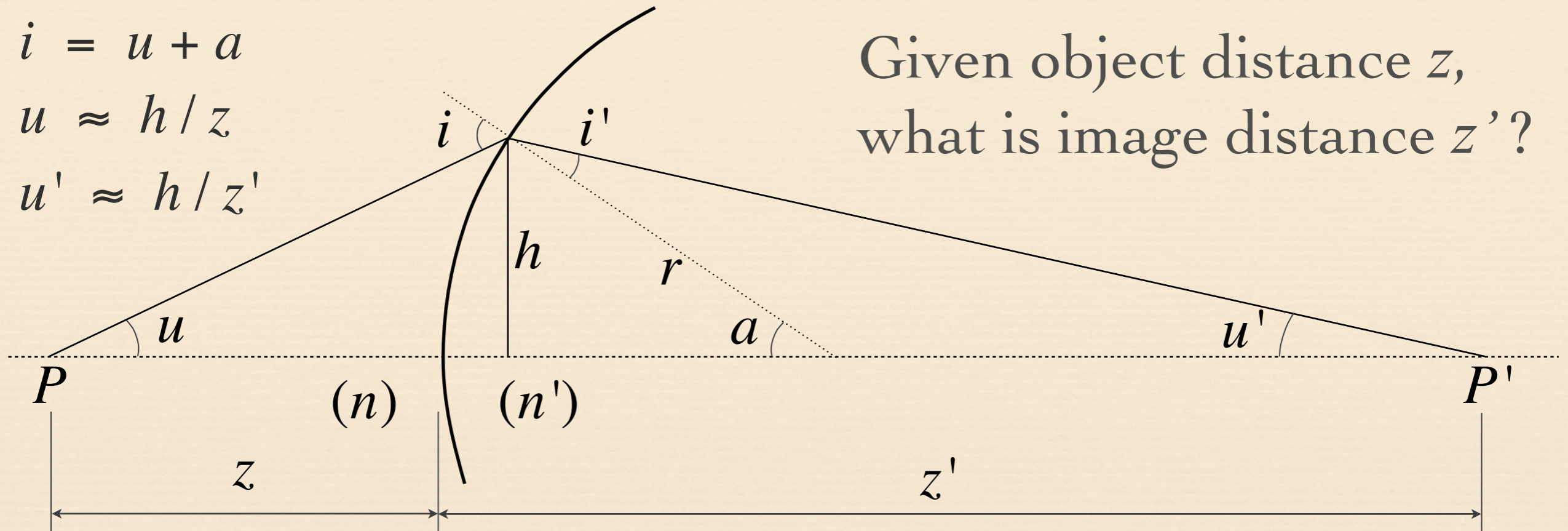
i, i' in radians

θ_i, θ_t in degrees

Paraxial focusing

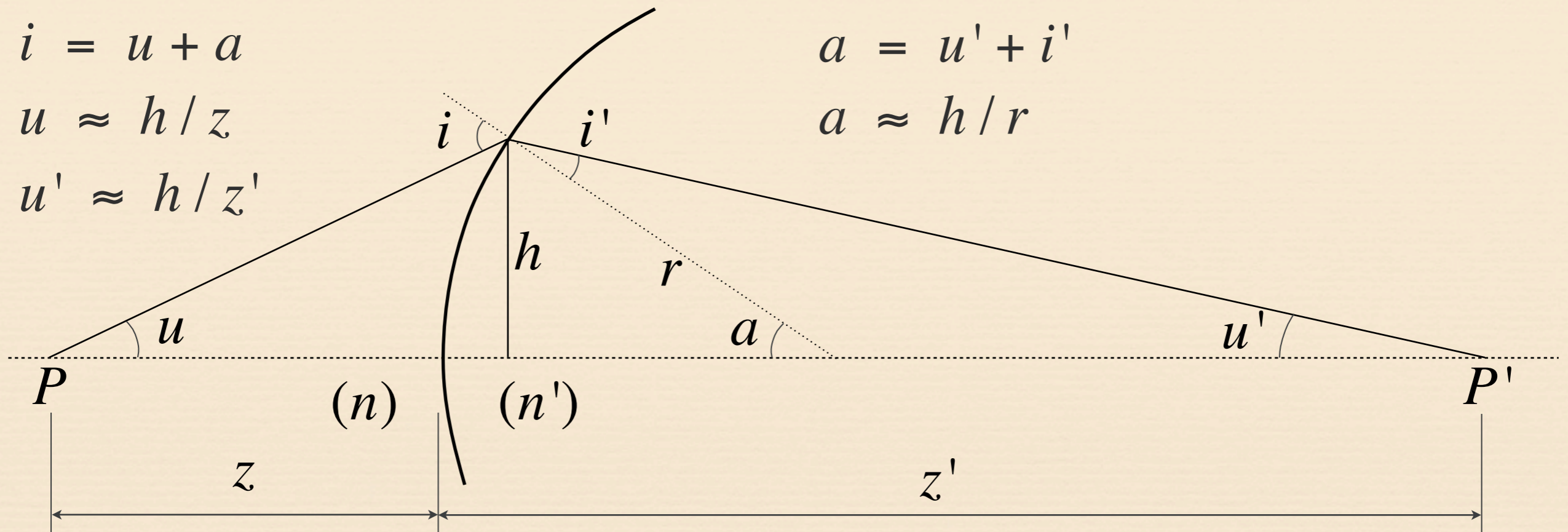
$$i = u + a$$
$$u \approx h/z$$
$$u' \approx h/z'$$

Given object distance z ,
what is image distance z' ?



$$n i \approx n' i'$$

Paraxial focusing



$$n(u + a) \approx n'(a - u')$$

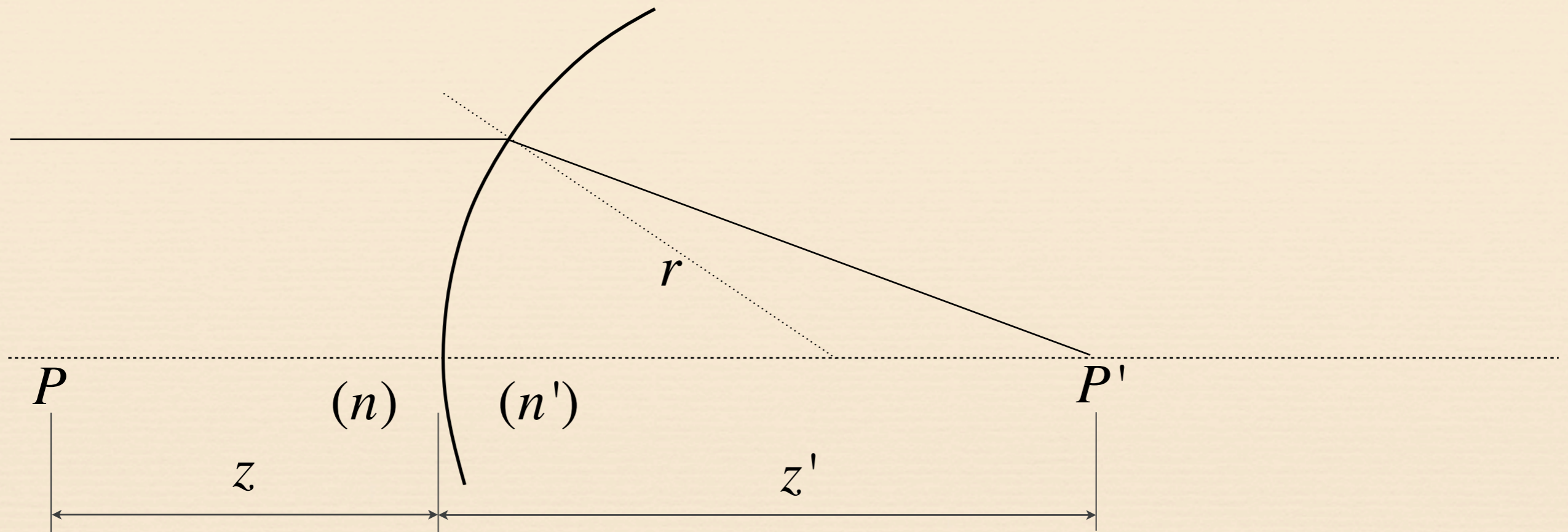
$$n(h/z + h/r) \approx n'(h/r - h/z')$$

$$n/z + n/r \approx n'/r - n'/z'$$

$$ni \approx n'i'$$

◆ h has canceled out, so any ray from P will focus to P'

Focal length



What happens if z is ∞ ?

$$n/z + n/r \approx n'/r - n'/z'$$

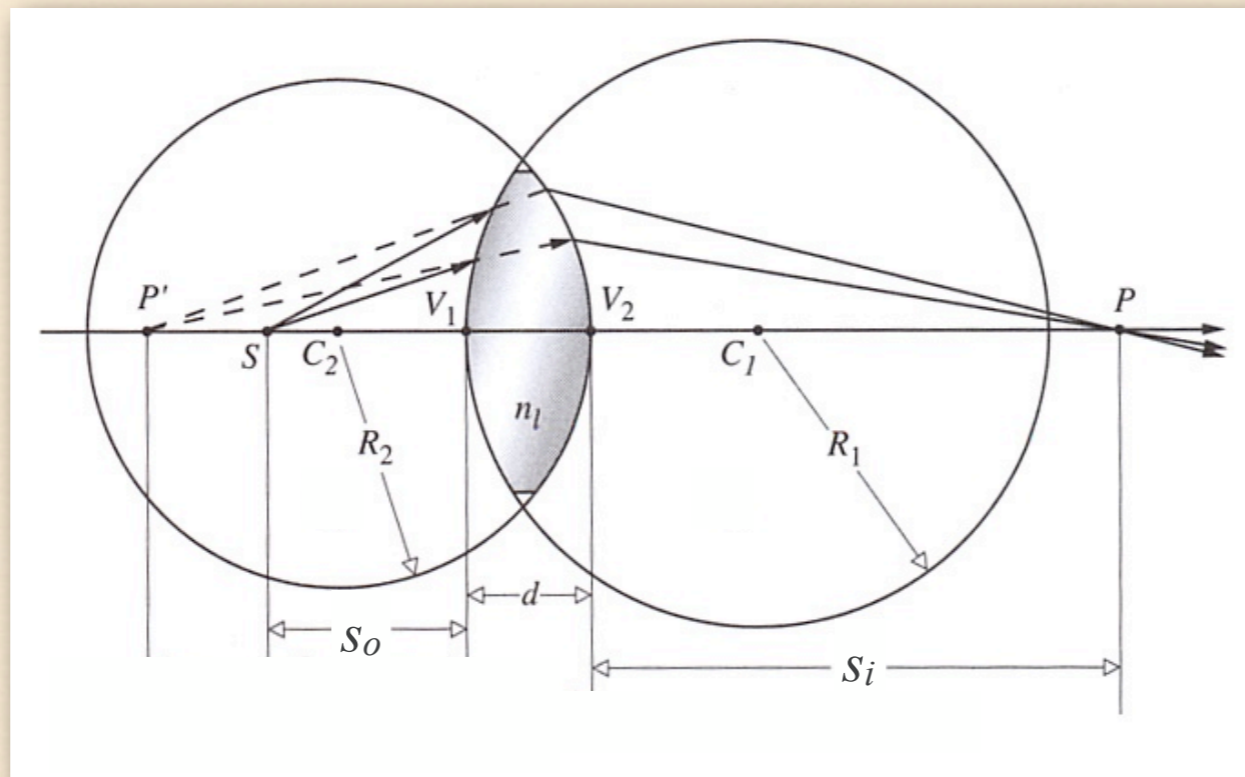
$$n/r \approx n'/r - n'/z'$$

$$z' \approx (r n') / (n' - n)$$

◆ $f \triangleq$ focal length = z'

Lensmaker's formula

- ◆ using similar derivations, one can extend these results to two spherical interfaces forming a lens in air



(Hecht, edited)

- ◆ as $d \rightarrow 0$ (*thin lens approximation*), we obtain the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Gaussian lens formula

- ◆ Starting from the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right), \quad (\text{Hecht, eqn 5.15})$$

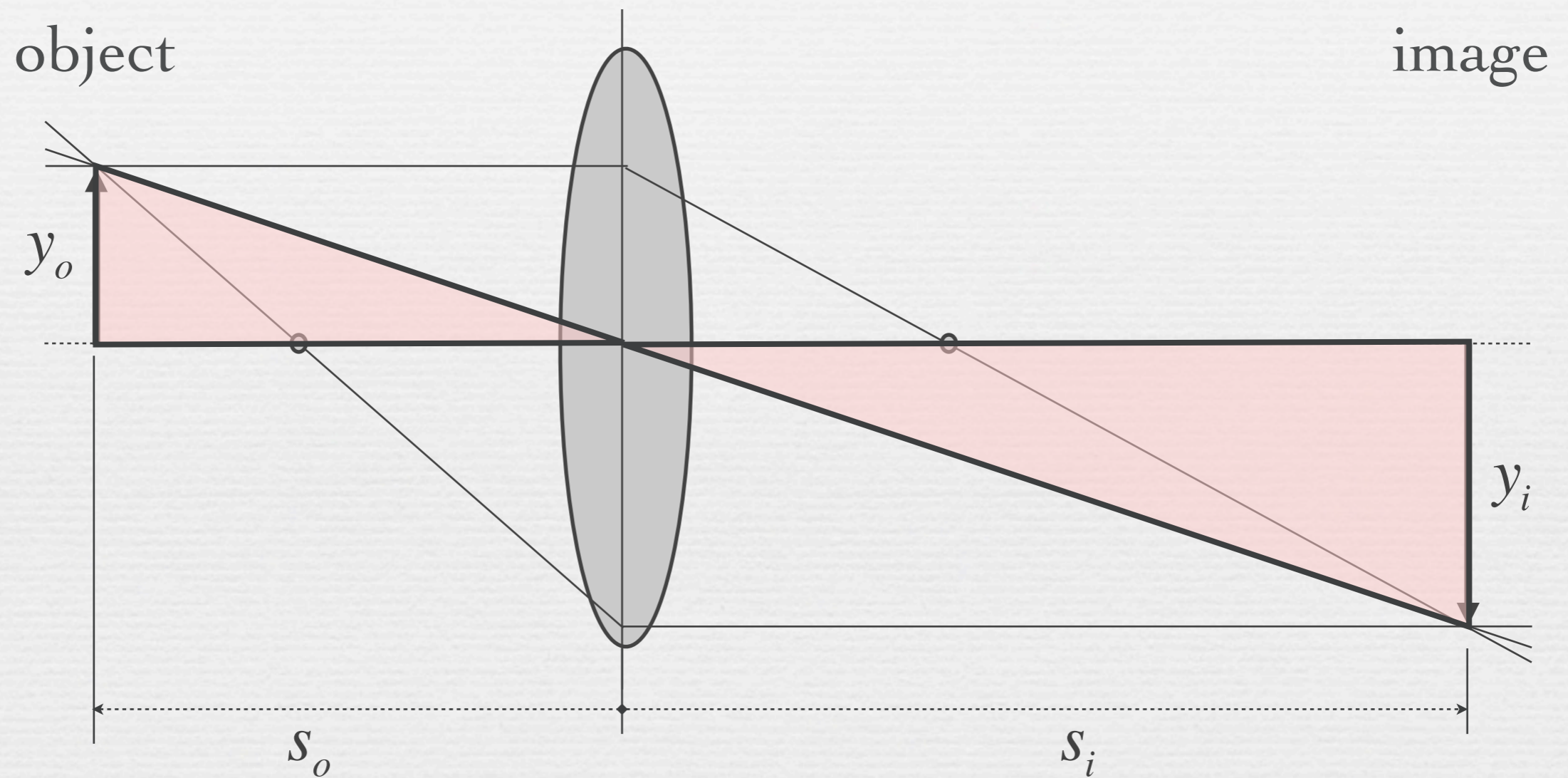
- ◆ and recalling that as object distance s_o is moved to infinity, image distance s_i becomes focal length f_i , we get

$$\frac{1}{f_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right). \quad (\text{Hecht, eqn 5.16})$$

- ◆ Equating these two, we get the Gaussian lens formula

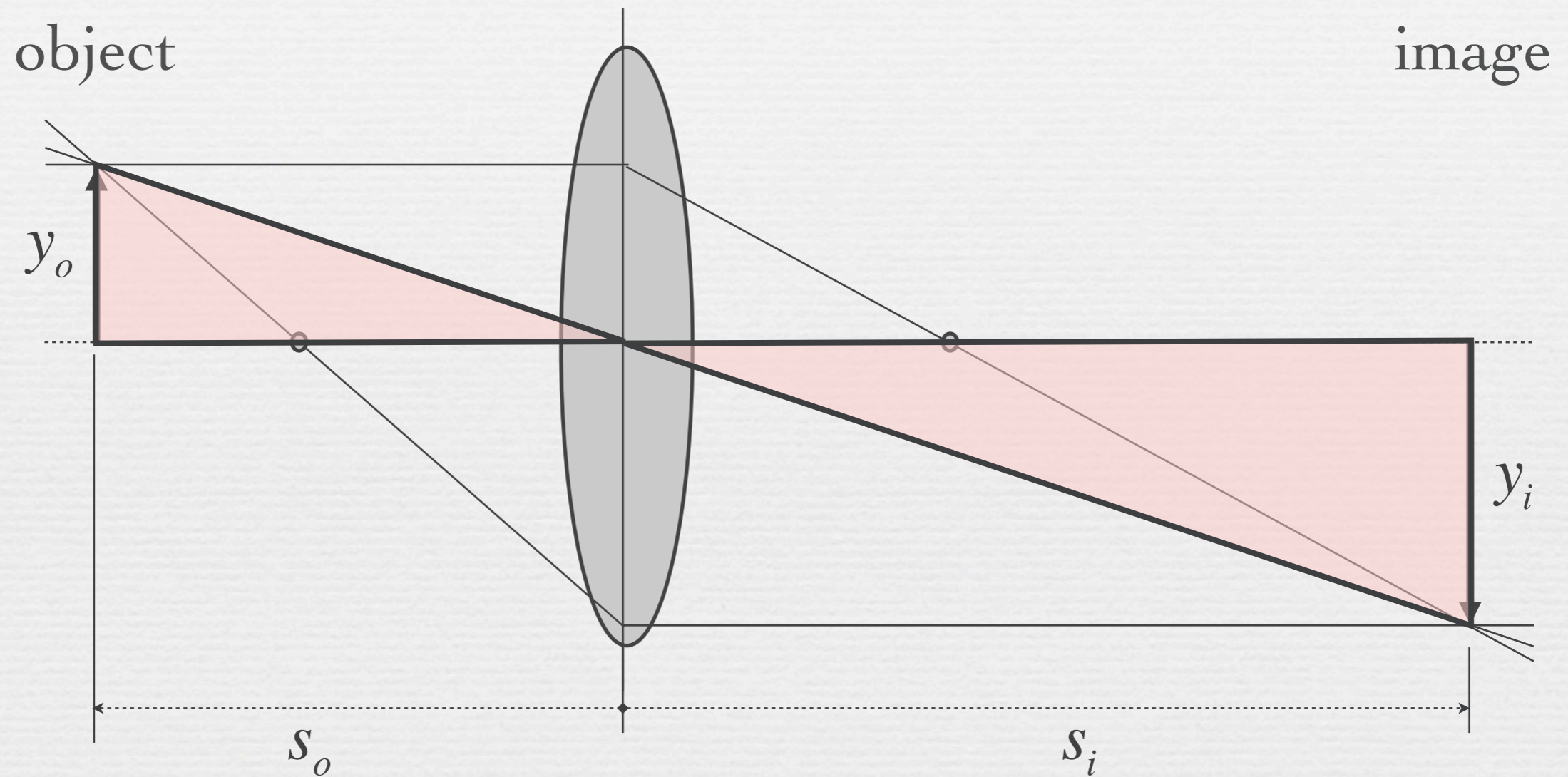
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f_i}. \quad (\text{Hecht, eqn 5.17})$$

From Gauss's ray construction to the Gaussian lens formula



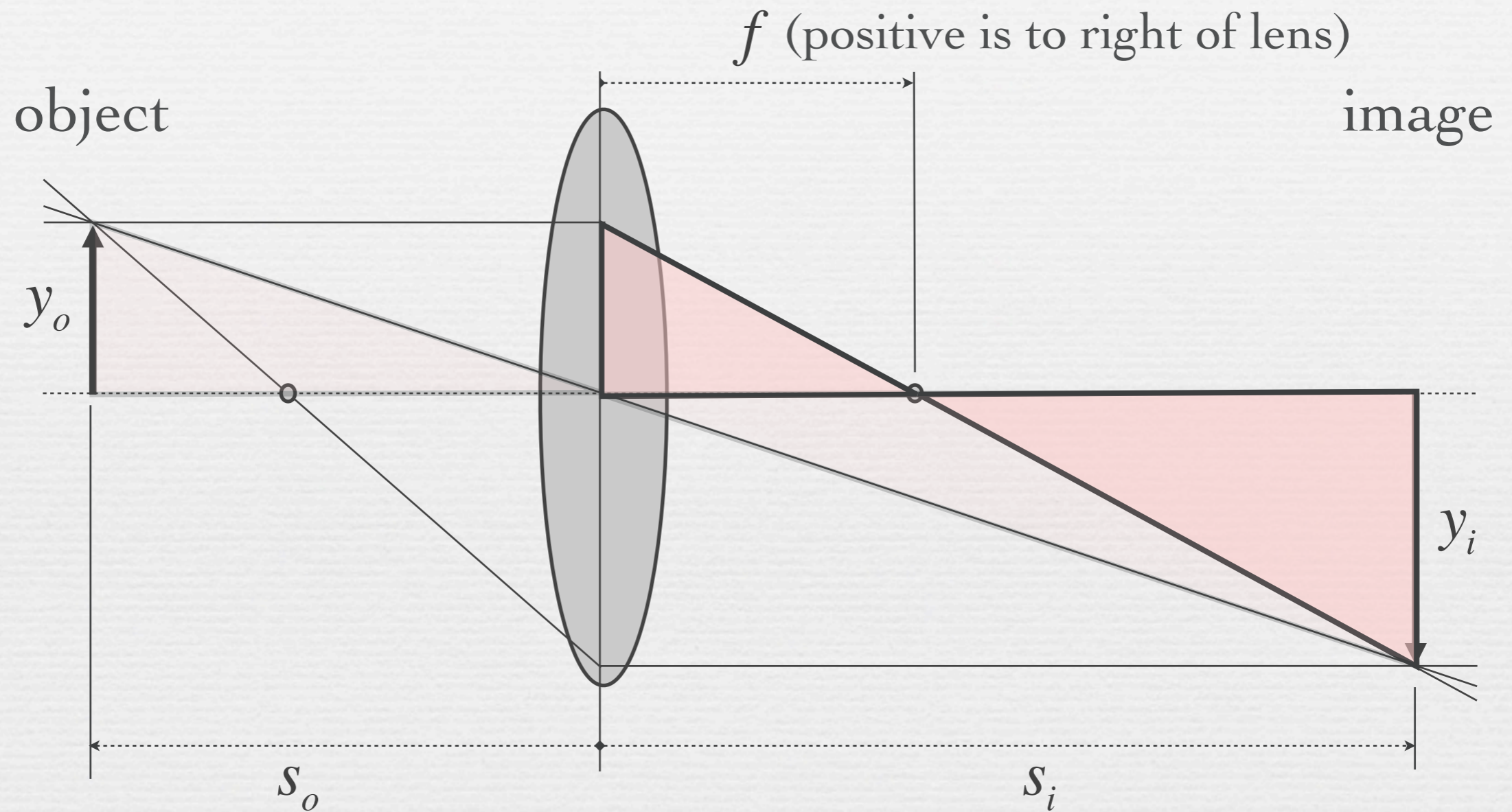
- ◆ positive s_i is rightward, positive s_o is leftward
- ◆ positive y is upward

From Gauss's ray construction to the Gaussian lens formula



$$\frac{|y_i|}{y_o} = \frac{s_i}{s_o}$$

From Gauss's ray construction to the Gaussian lens formula

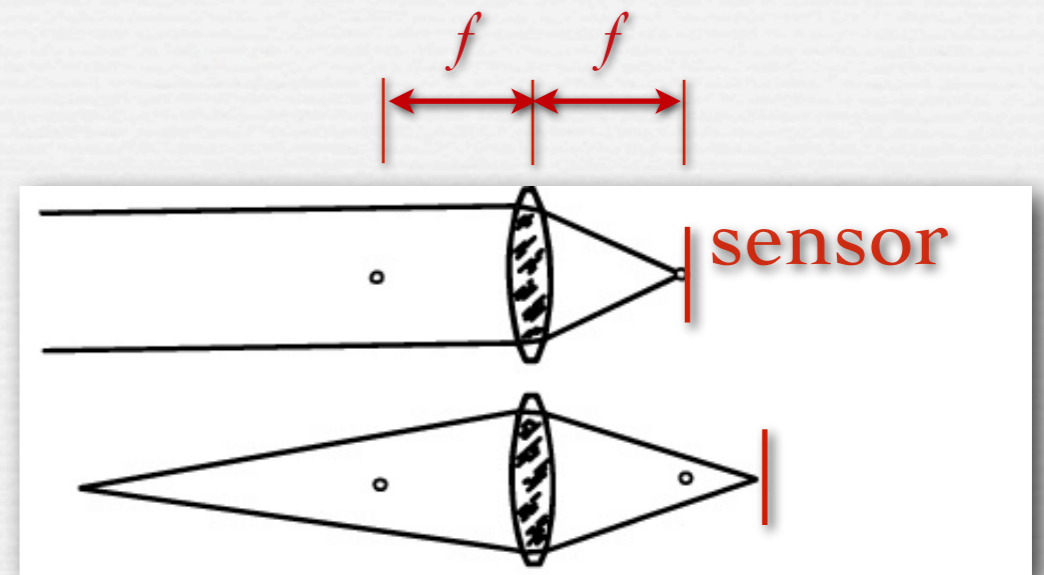


$$\frac{|y_i|}{y_o} = \frac{s_i}{s_o} \quad \text{and} \quad \frac{|y_i|}{y_o} = \frac{s_i - f}{f} \quad \dots$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Changing the focus distance

- ◆ to focus on objects at different distances, move sensor relative to lens



(FLASH DEMO)

<http://graphics.stanford.edu/courses/cs178/applets/gaussian.html>

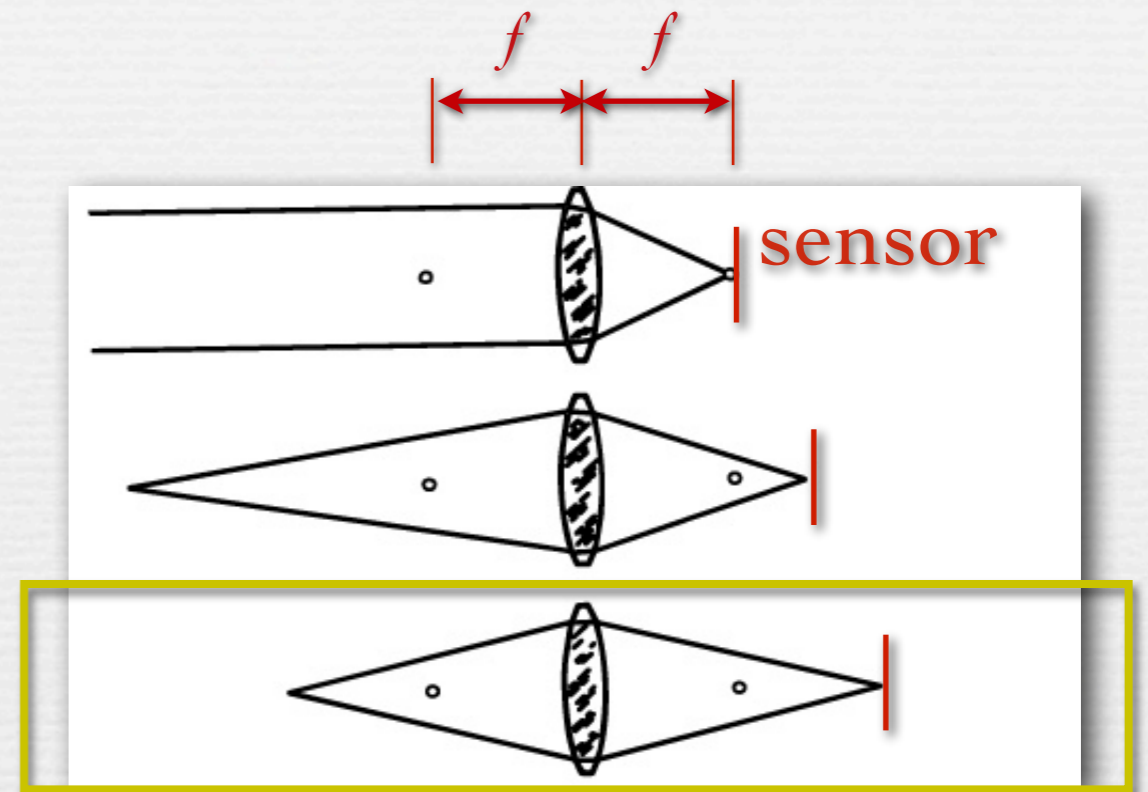
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Changing the focus distance

- ◆ to focus on objects at different distances, move sensor relative to lens
- ◆ at $s_o = s_i = 2f$ we have 1:1 imaging, because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

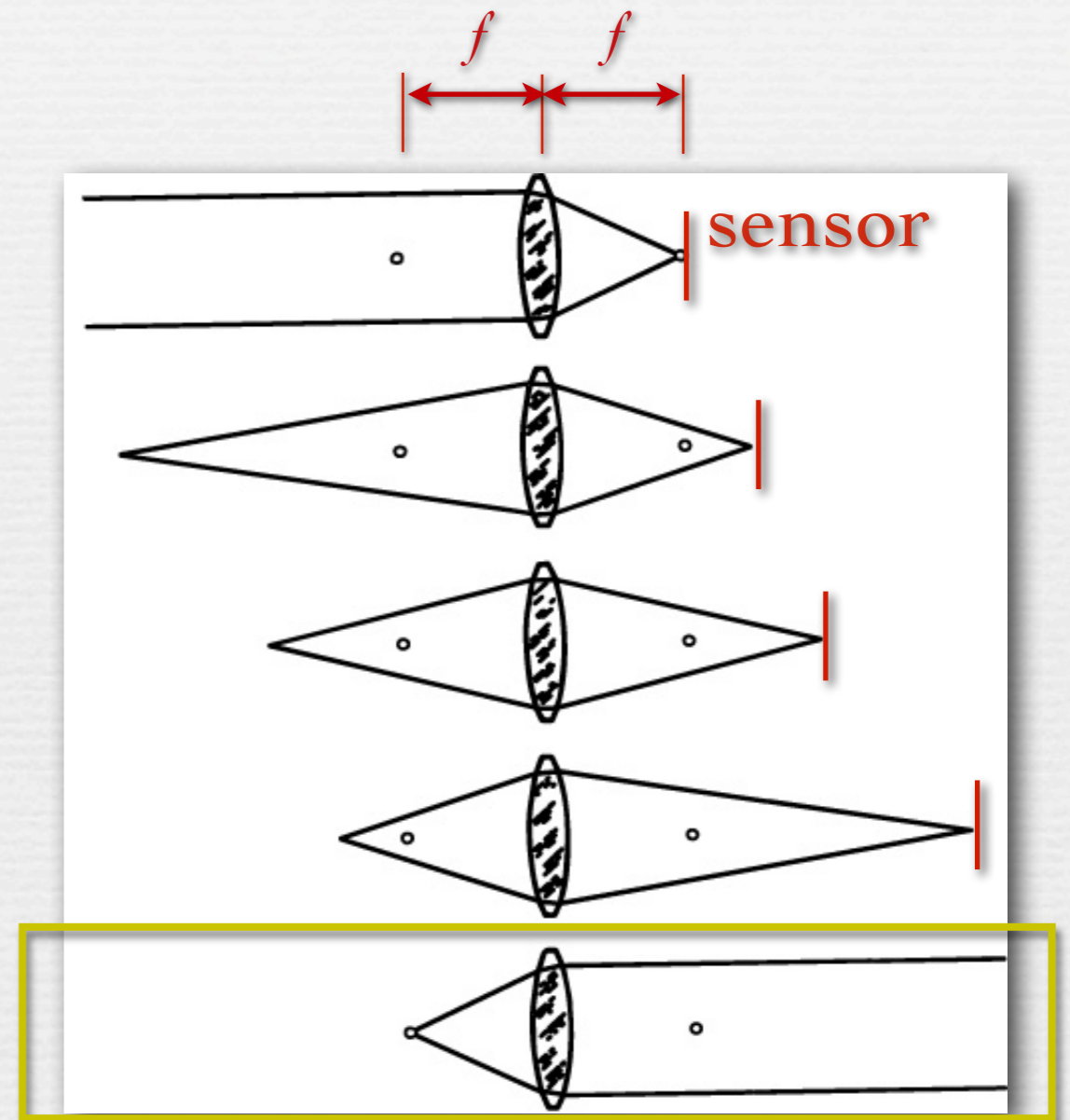
In 1:1 imaging, if the sensor is 36mm wide, an object 36mm wide will fill the frame.



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Changing the focus distance

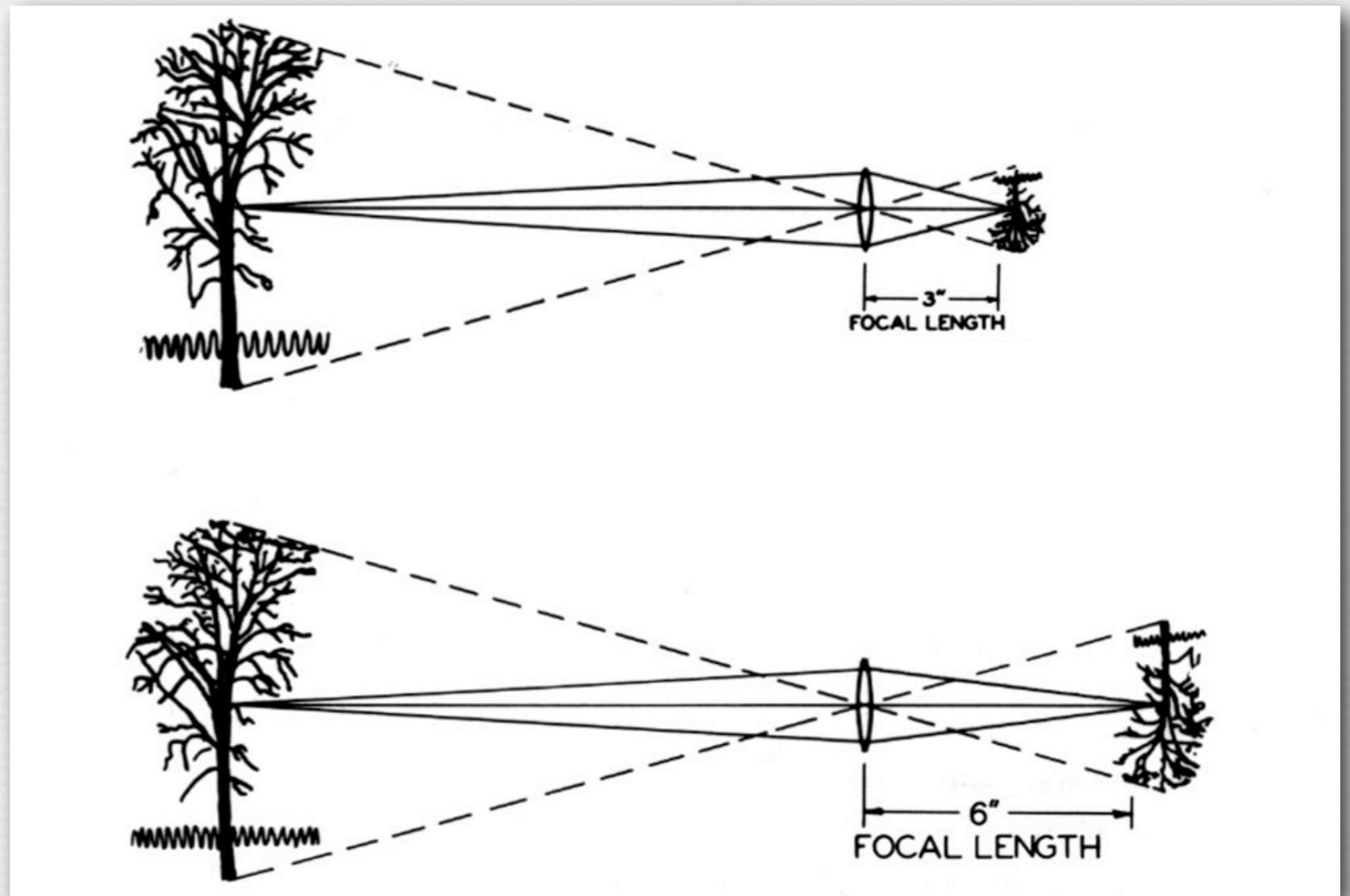
- ◆ to focus on objects at different distances, move sensor relative to lens
- ◆ at $s_o = s_i = 2f$ we have 1:1 imaging, because
$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$
- ◆ can't focus on objects closer to lens than its focal length f



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Changing the focal length

- ◆ weaker lenses have longer focal lengths
- ◆ to stay in focus, move the sensor further back
- ◆ focused image of tree is located slightly beyond the focal length

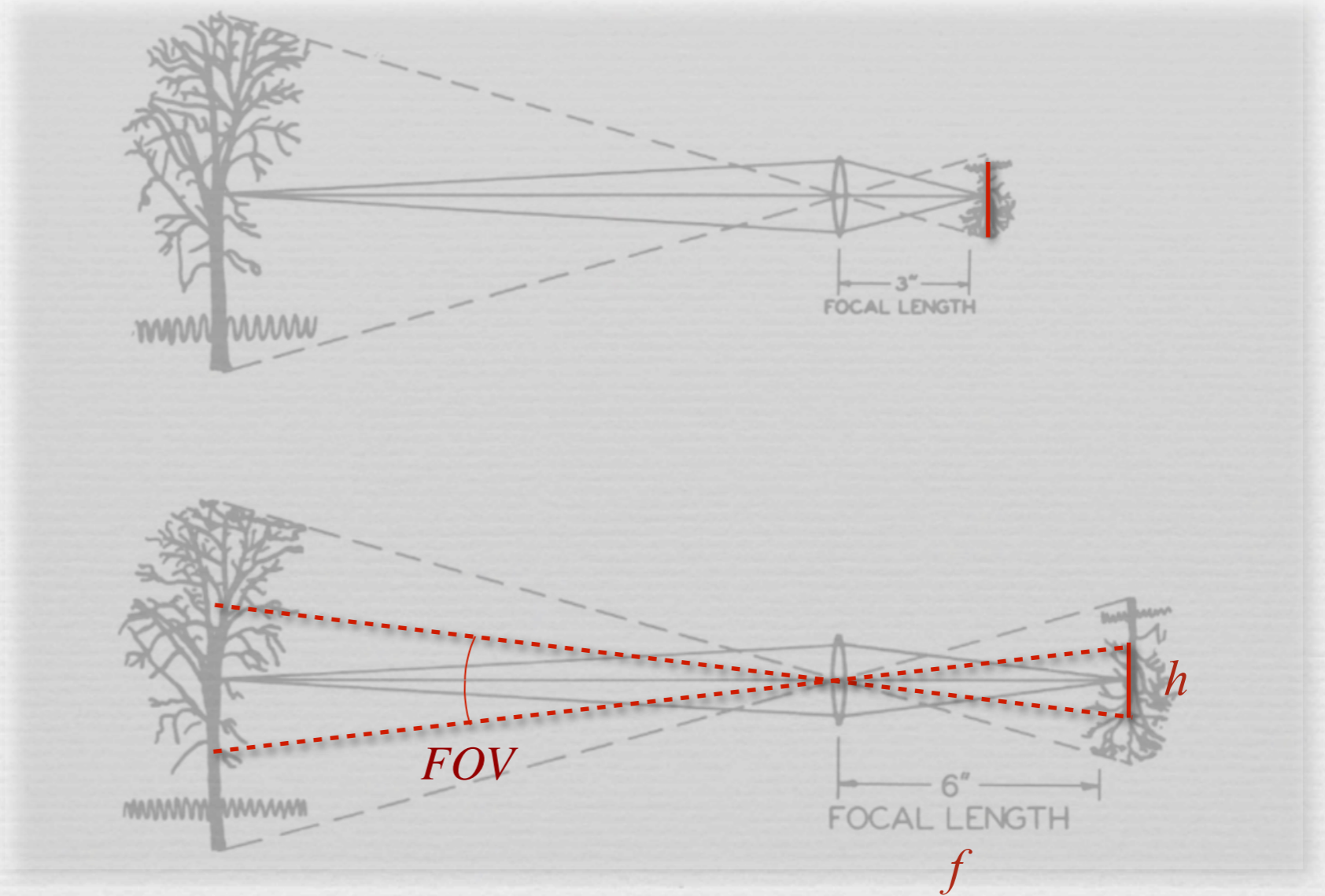


(Kingslake)

The tree would be in focus at the lens focal length only if it were infinitely far away.

Changing the focal length

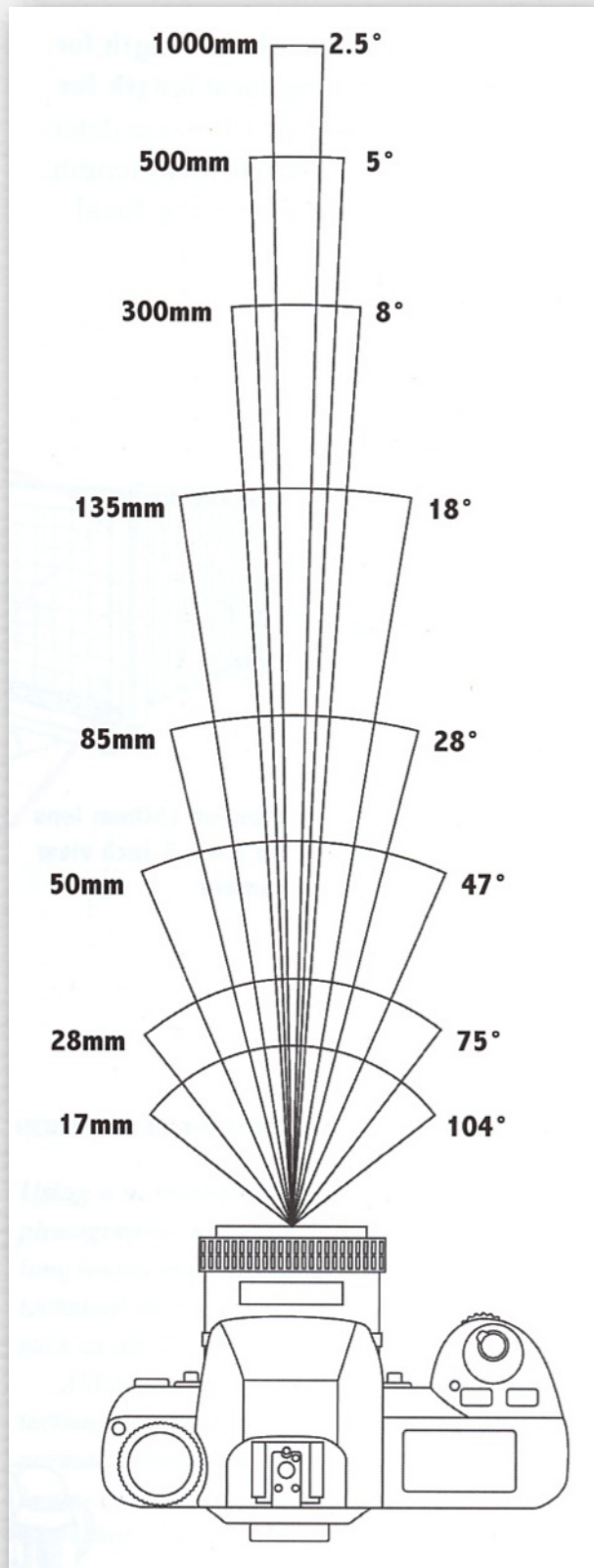
- ◆ if the sensor size is constant, the field of view becomes smaller



(Kingslake)

$$FOV = 2 \arctan (h / 2f)$$

Focal length and field of view



17mm



28mm



50mm

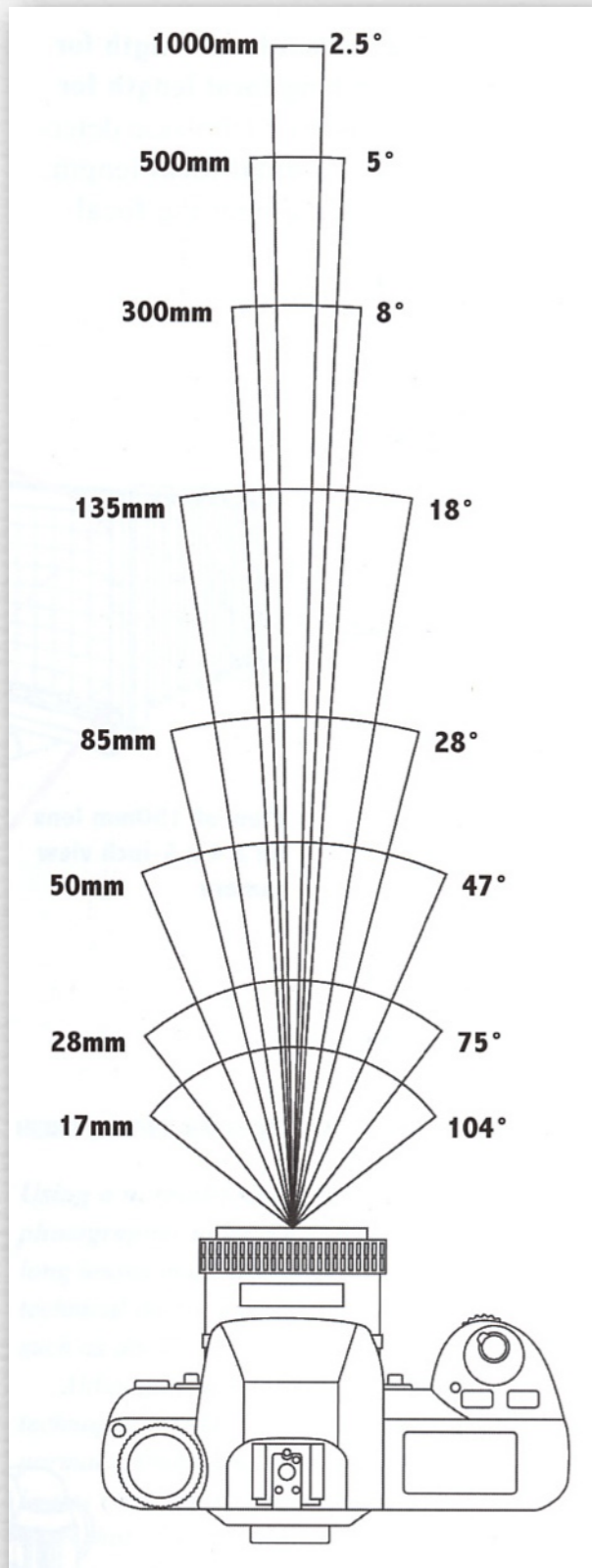


85mm

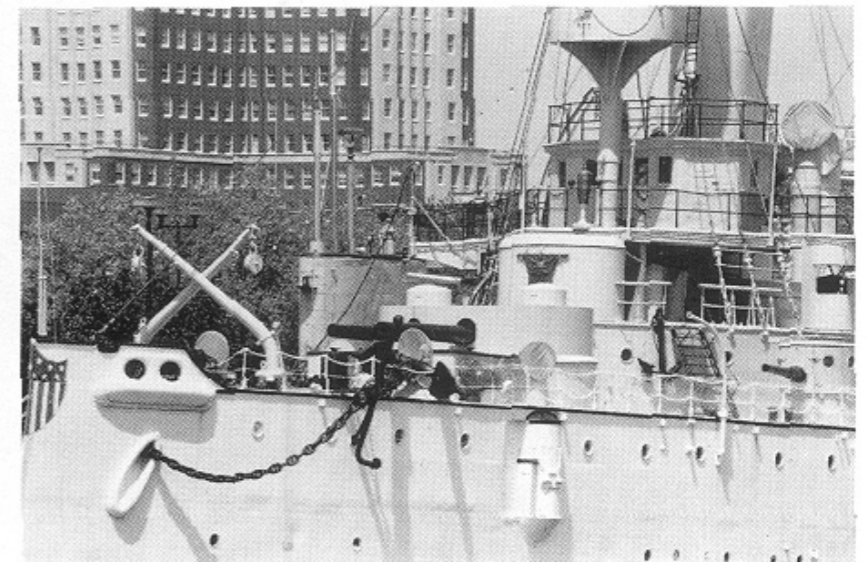
(London)

FOV measured diagonally on a 35mm full-frame camera (24 × 36mm)

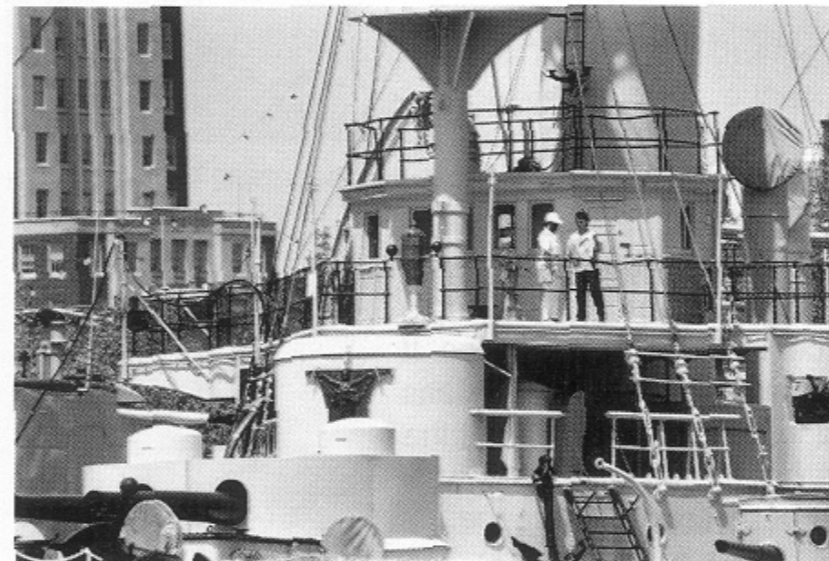
Focal length and field of view



135mm



300mm



500mm



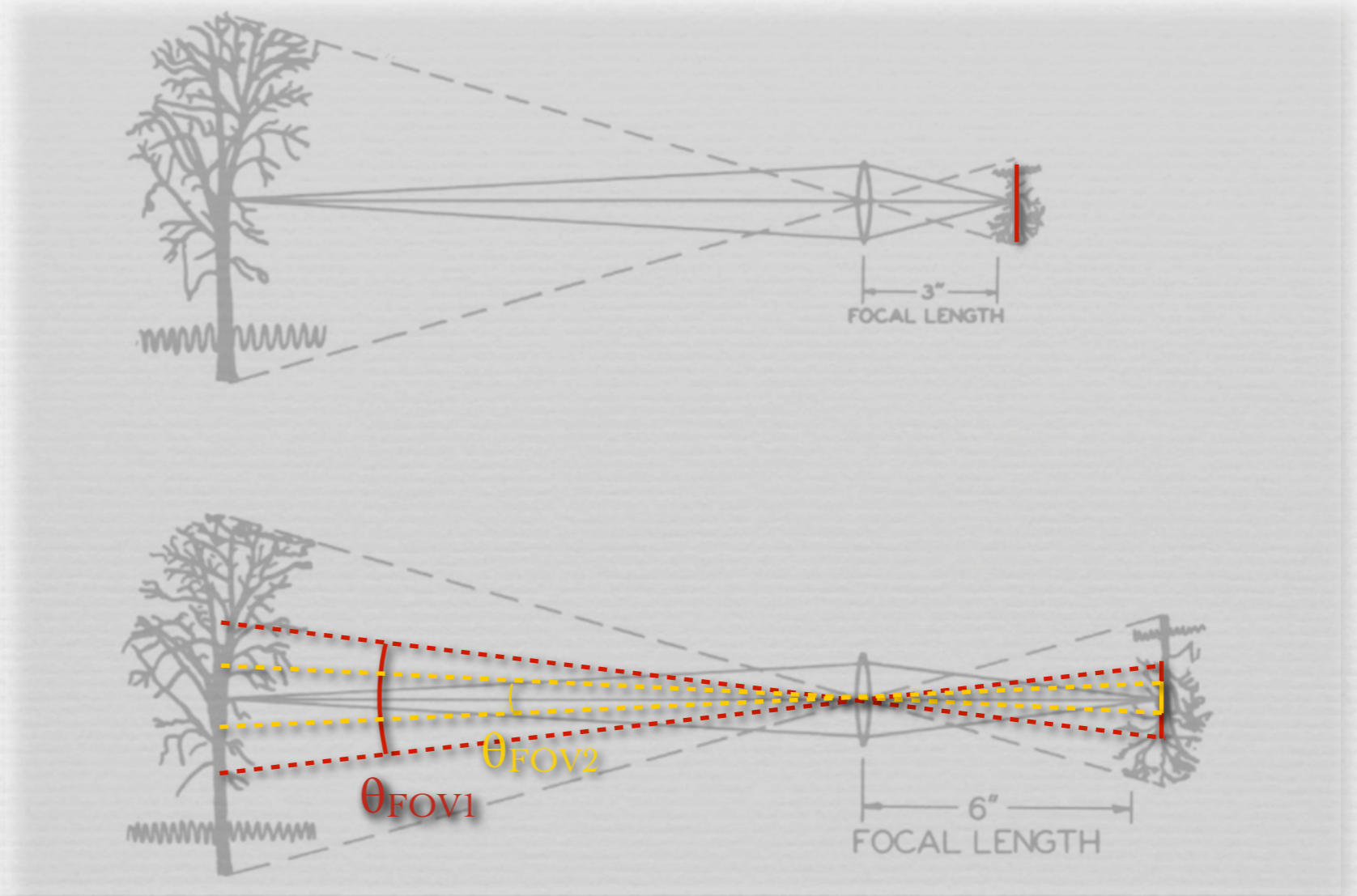
1000mm

(London)

FOV measured diagonally on a 35mm full-frame camera (24 × 36mm)

Changing the sensor size

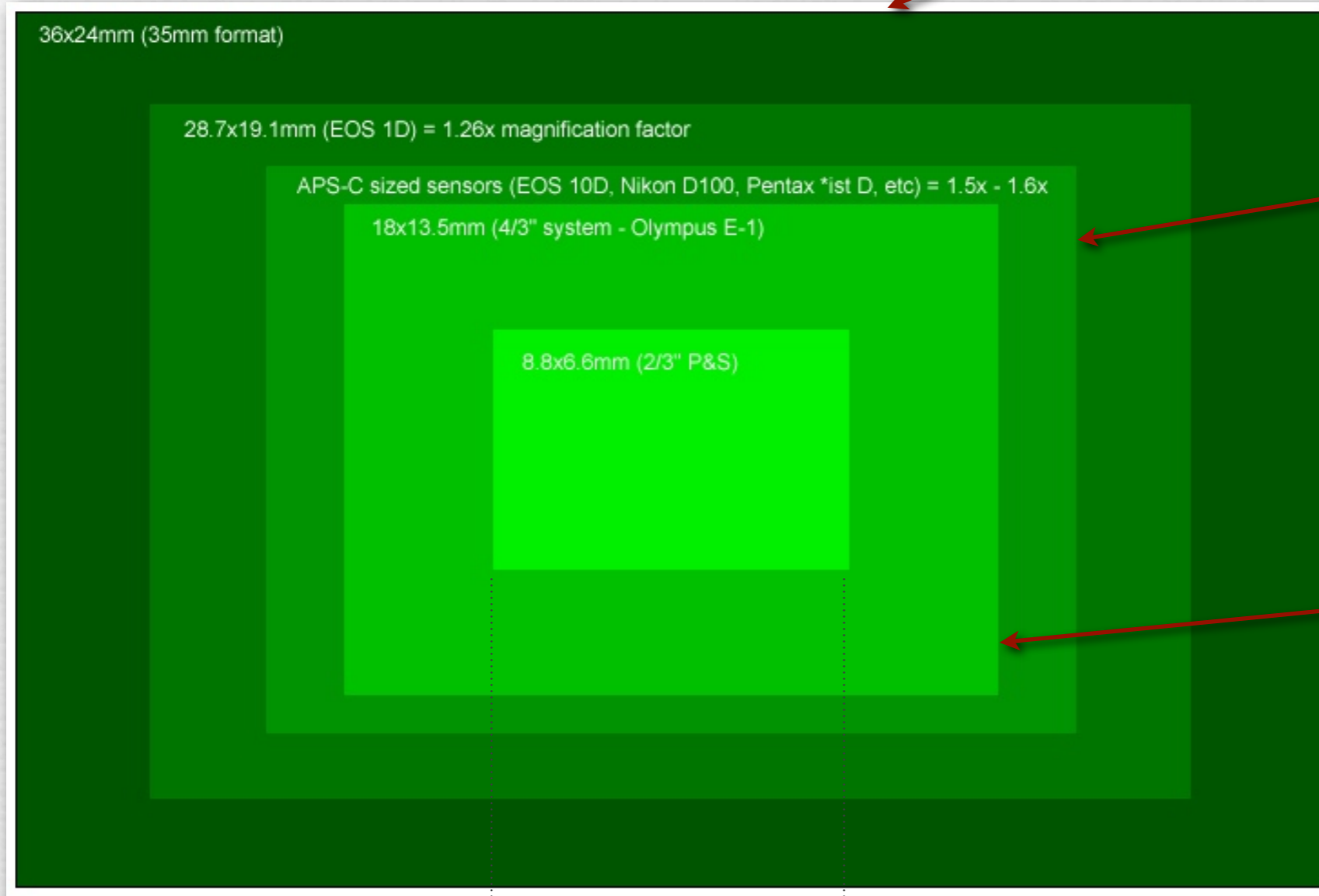
- ◆ if the sensor size is smaller, the field of view is smaller too
- ◆ smaller sensors either have fewer pixels, or noisier pixels



(Kingslake)

Sensor sizes

“full frame”
Canon 5D Mark II
(24mm × 36mm)



“APS-C”
Nikon D40
(15.5mm × 23.7mm)
(~1.5× crop factor)

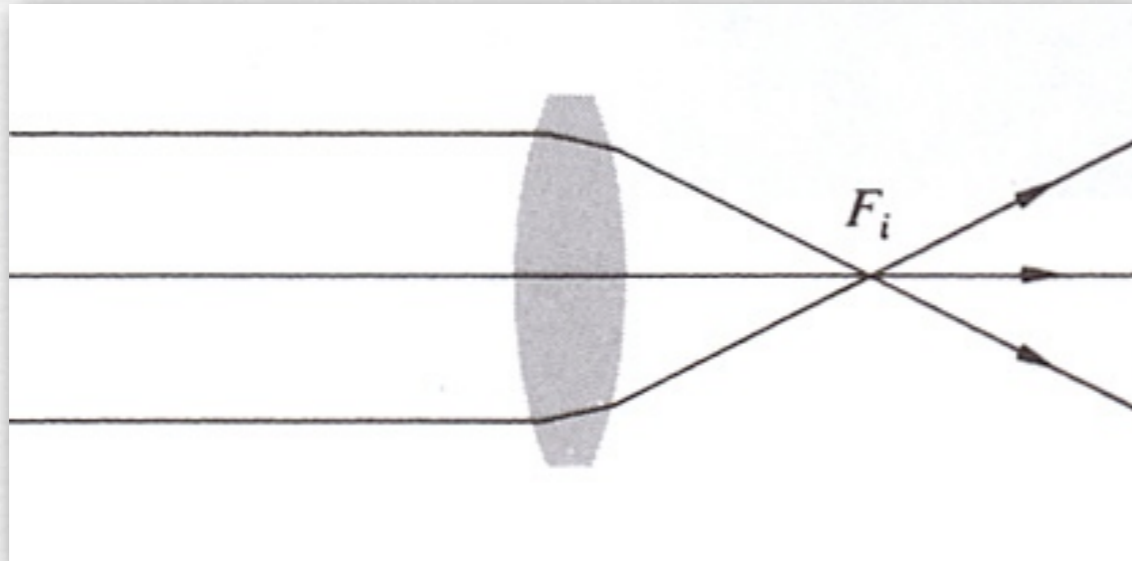
“micro 4/3”
Panasonic GF1
(13mm × 17.3mm)
(~2× crop factor)

“point-and-shoot”
Canon A590
(5.75mm × 4.31mm)
(~8× crop factor)

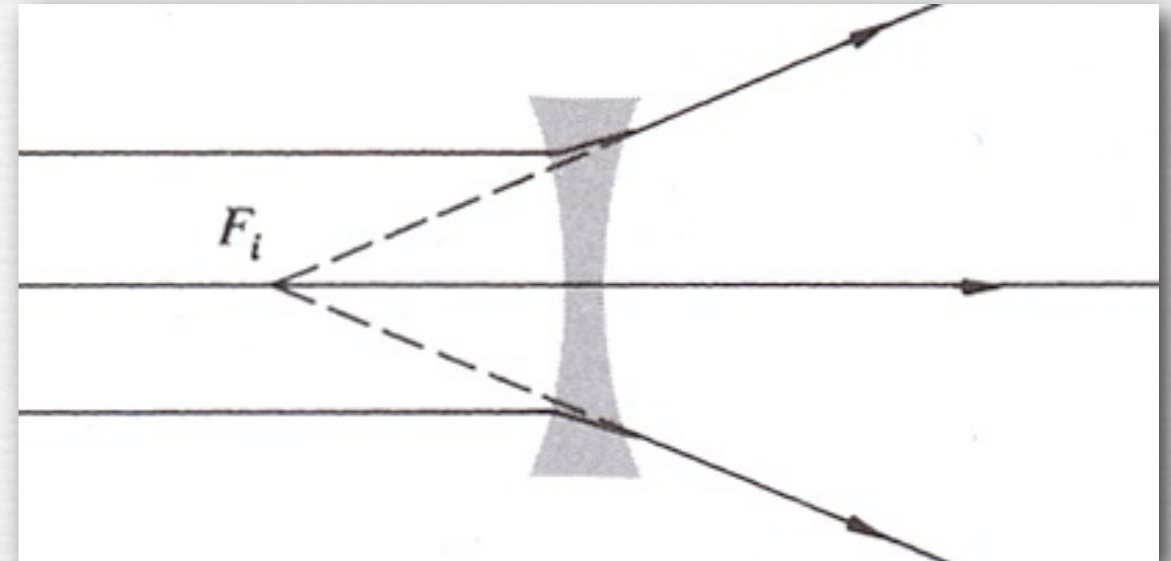


Convex versus concave lenses

(Hecht)



rays from a convex lens converge

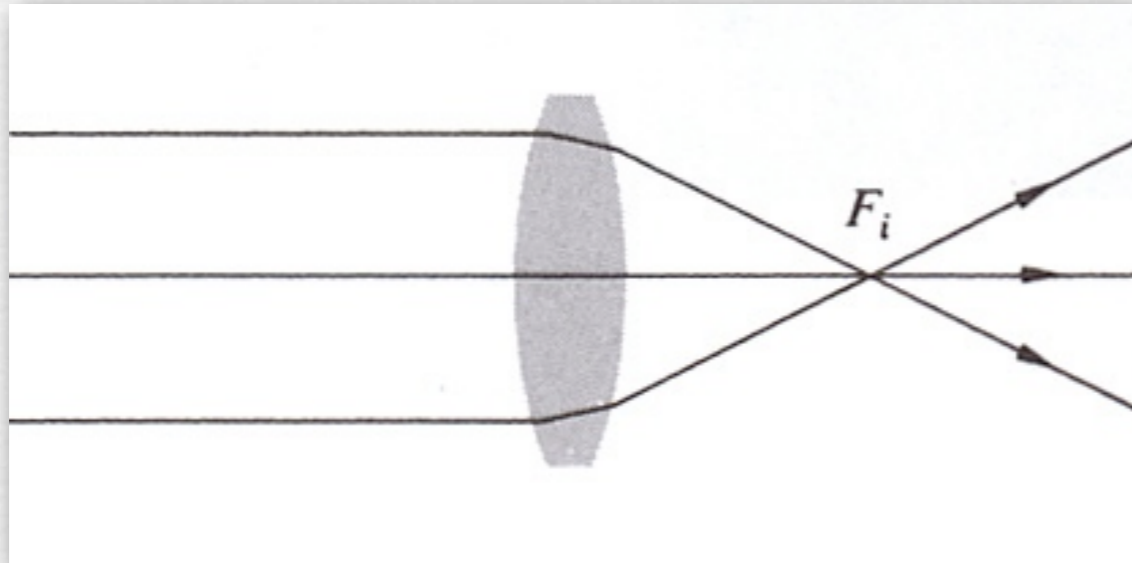


rays from a concave lens diverge

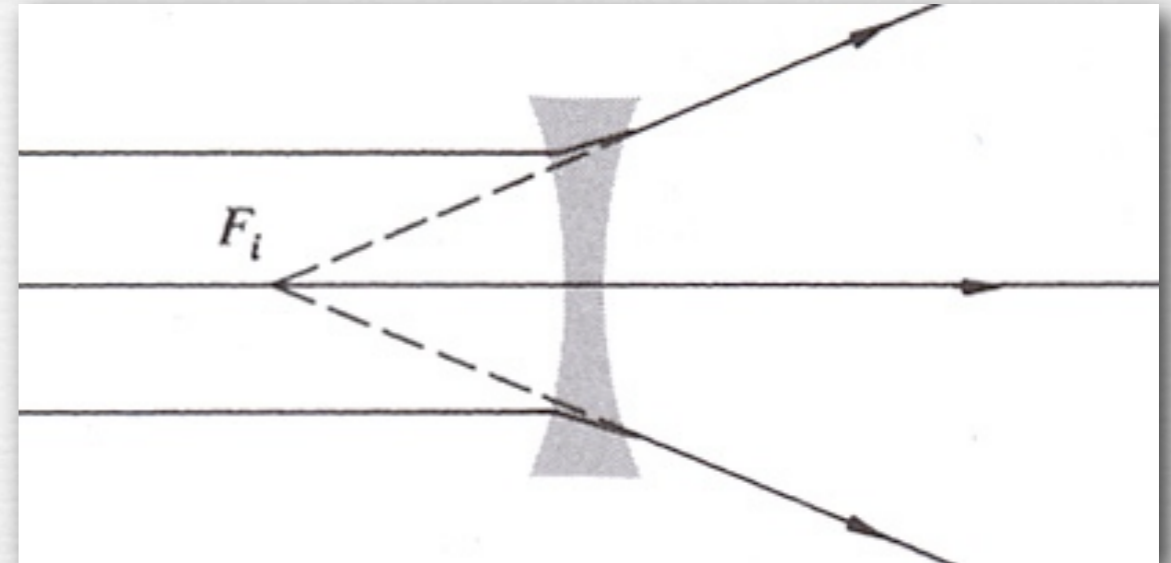
- ◆ positive focal length f means parallel rays from the left converge to a point on the right
- ◆ negative focal length f means parallel rays from the left converge to a point on the left (dashed lines above)

Convex versus concave lenses

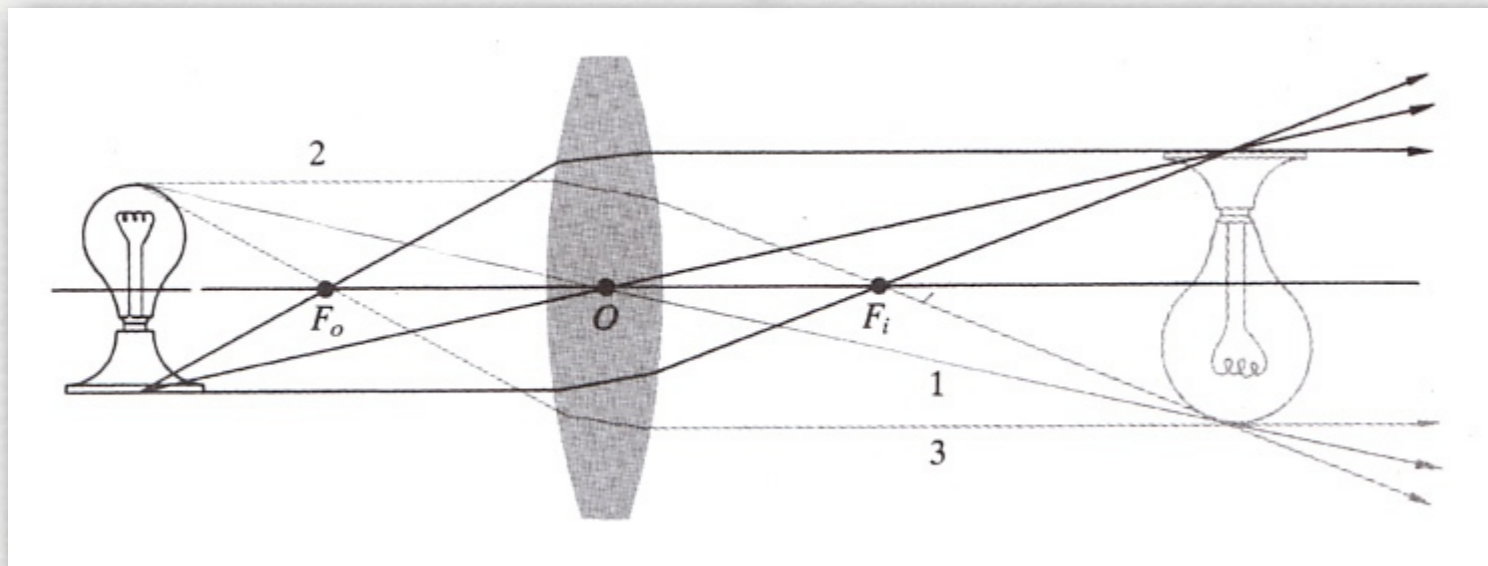
(Hecht)



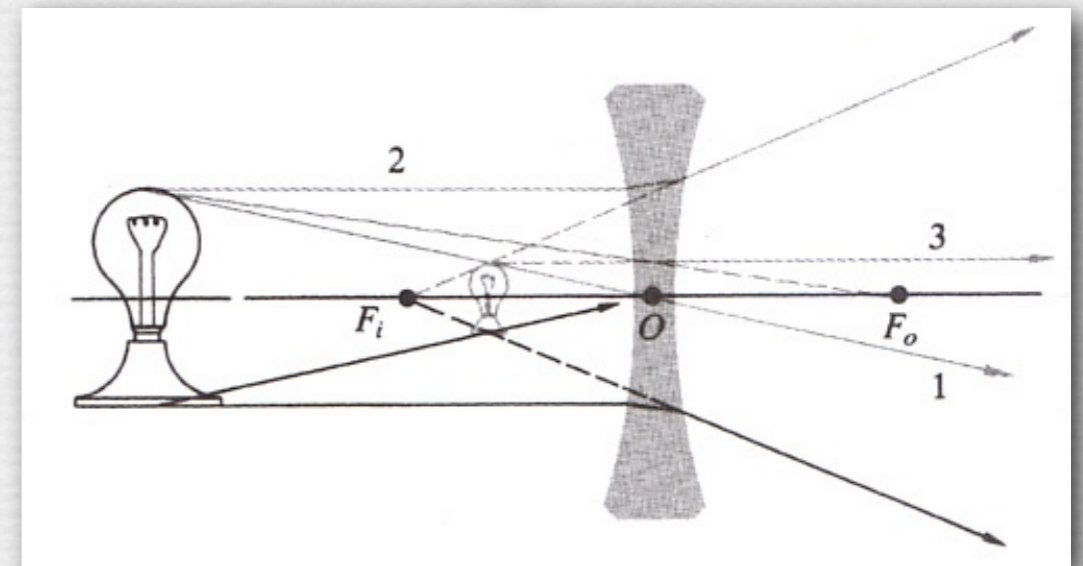
rays from a convex lens converge



rays from a concave lens diverge

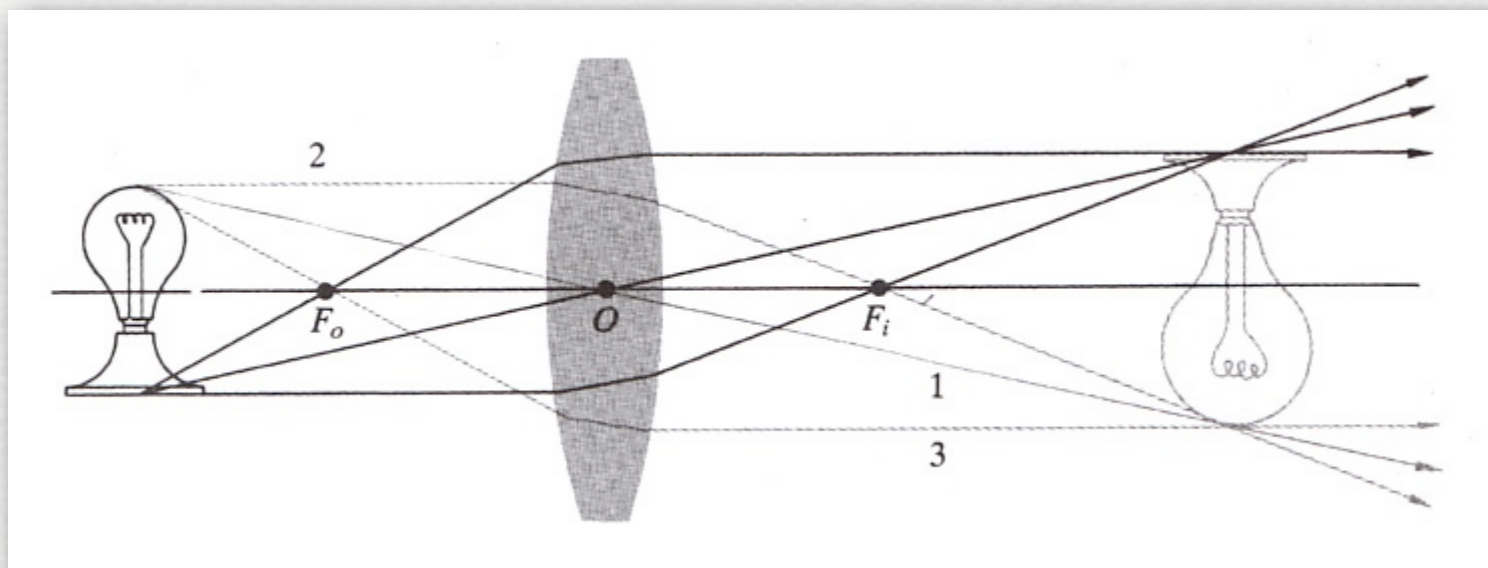
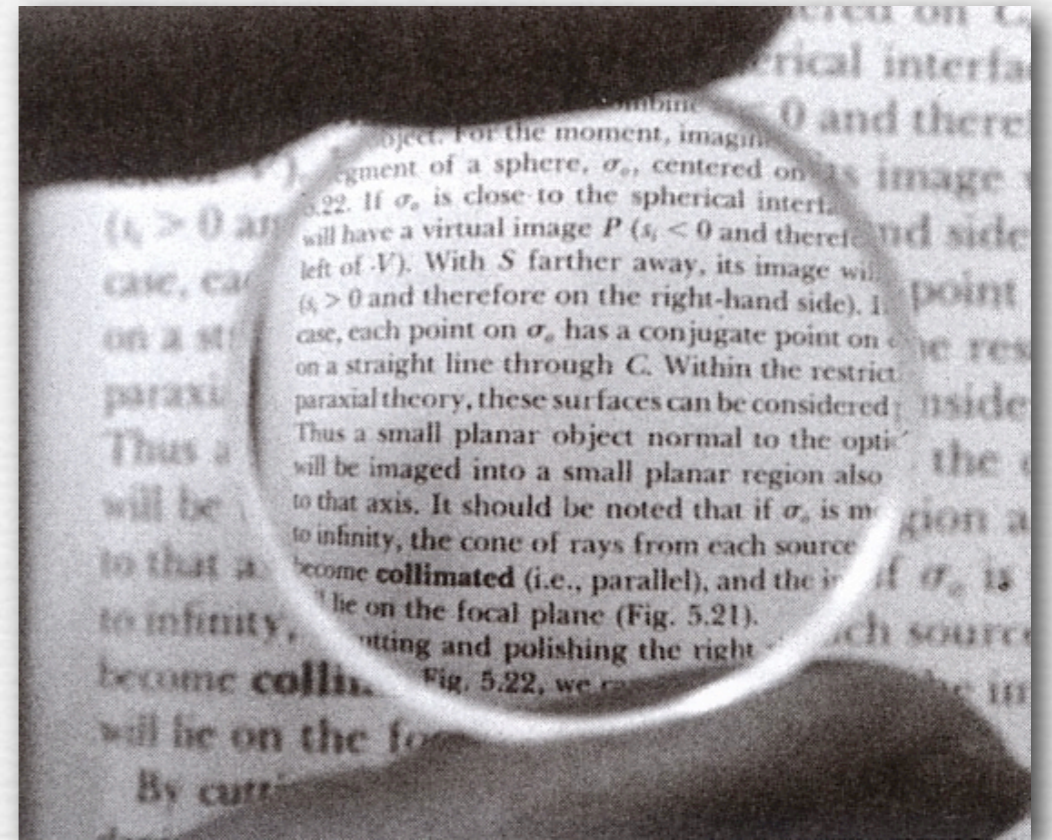
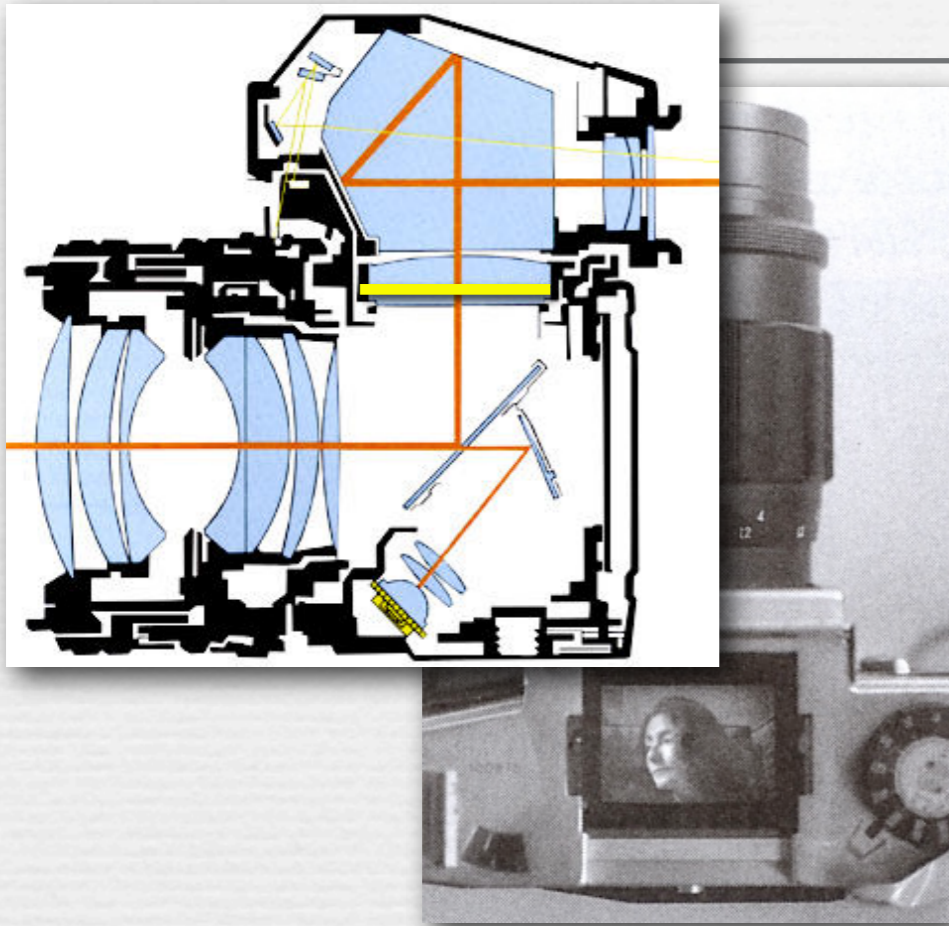


...producing a real image

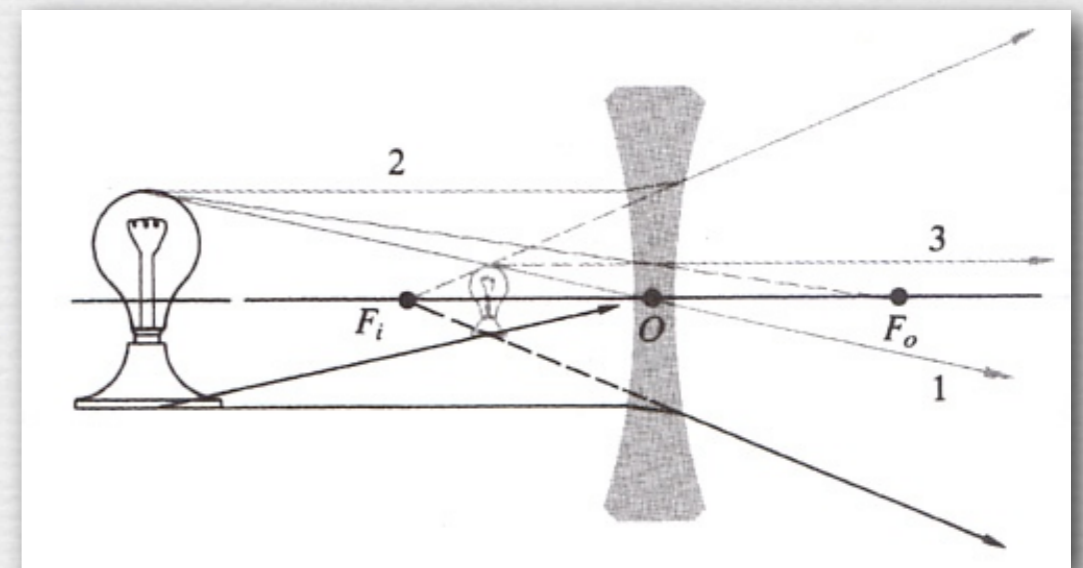


...producing a virtual image

Convex versus concave lenses

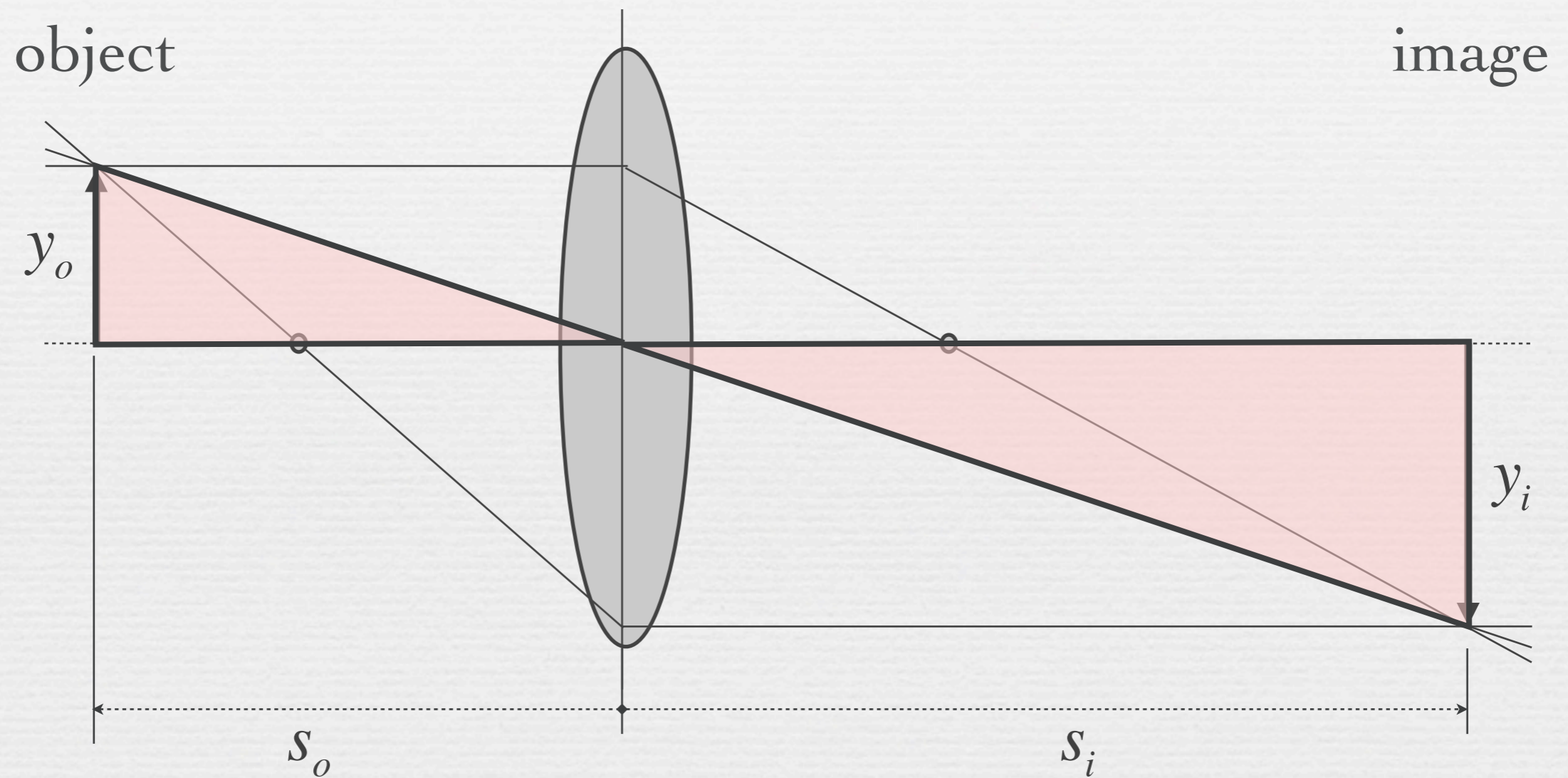


...producing a real image



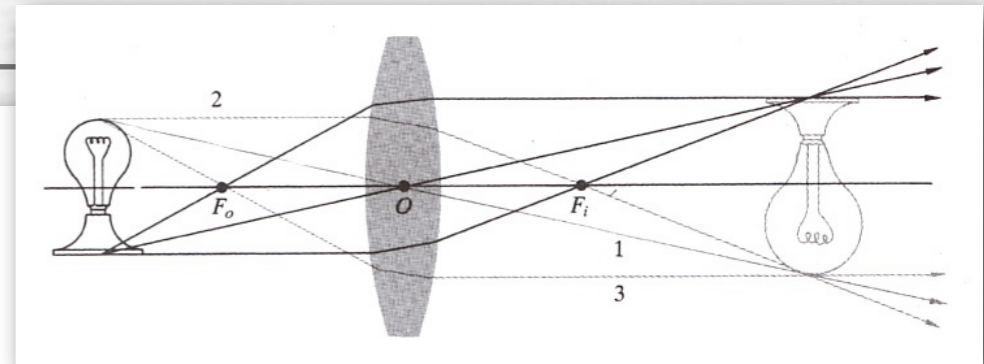
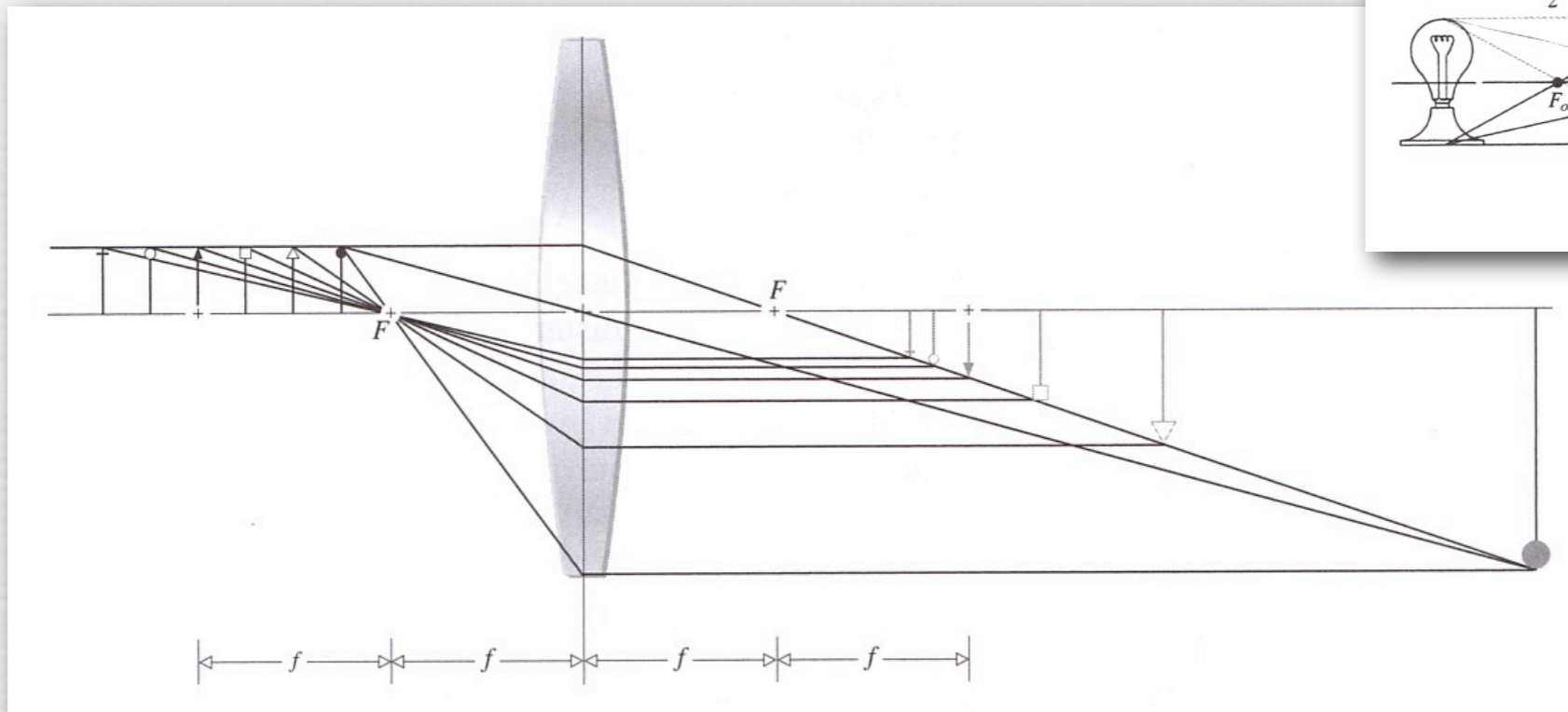
...producing a virtual image

Magnification



$$M_T @ \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

Lenses perform a 3D perspective transform



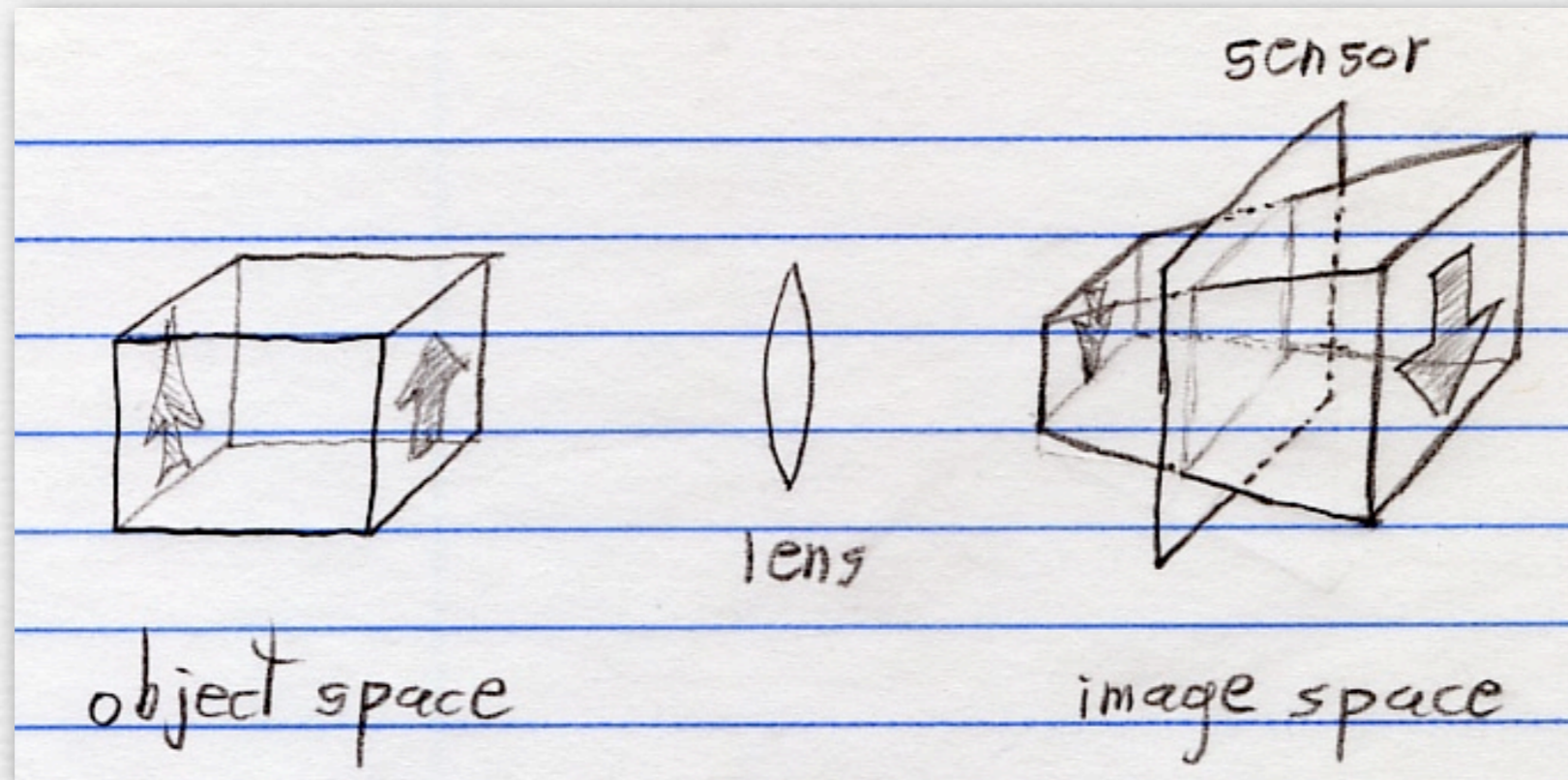
(FLASH DEMO)

<http://graphics.stanford.edu/courses/cs178/applets/thinlens.html>

(Hecht)

- ◆ lenses transform a 3D object to a 3D image; the sensor extracts a 2D slice from that image
- ◆ as an object moves linearly (in Z), its image moves non-proportionately (in Z)
- ◆ as you move a lens linearly relative to the sensor, the in-focus object plane moves non-proportionately
- ◆ as you refocus a camera, the image changes size !

Lenses perform a 3D perspective transform (contents of whiteboard)



- ◆ a cube in object space is transformed by a lens into a 3D frustum in image space, with the orientations shown by the arrows
- ◆ in computer graphics this transformation is modeled as a 4×4 matrix multiplication of 3D points expressed in 4D homogenous coordinates
- ◆ in photography a sensor extracts a 2D slice from the 3D frustum; on this slice some objects may be sharply focused; others may be blurry

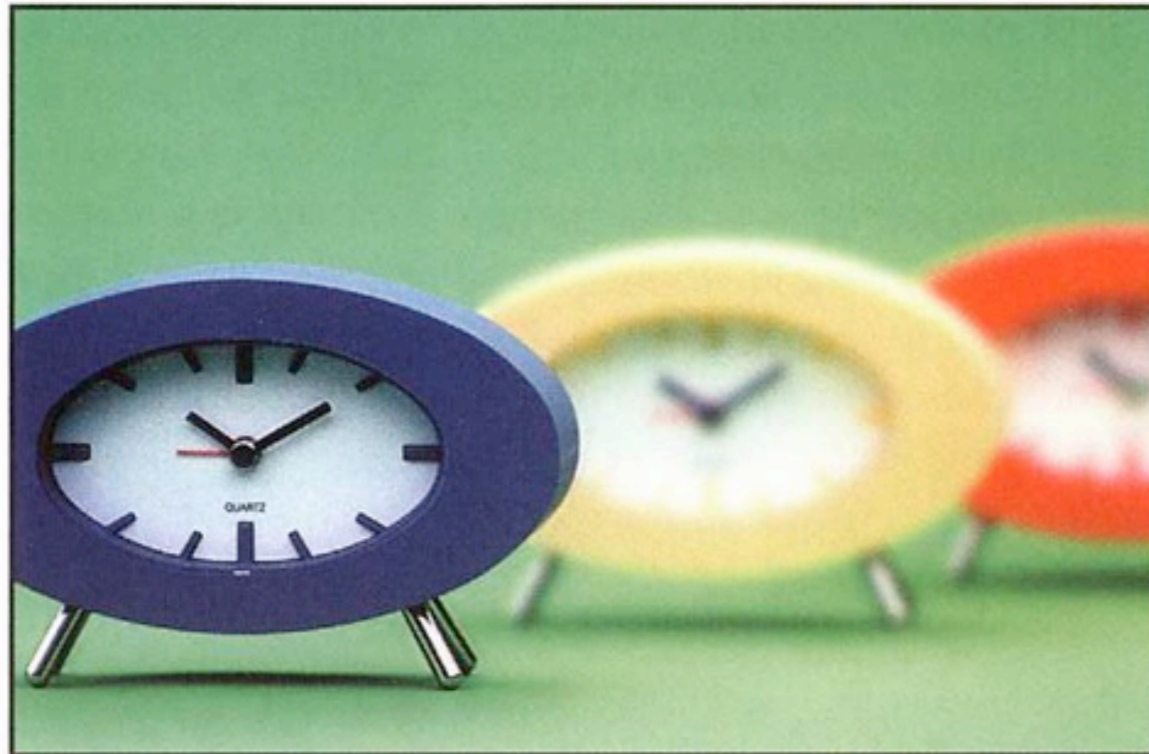
Recap

- ◆ approximations we sometimes make when analyzing lenses
 - geometrical optics instead of physical optics
 - spherical lenses instead of hyperbolic lenses
 - thin lens representation of thick optical systems
 - paraxial approximation of ray angles
- ◆ the Gaussian lens formula relates focal length, object distance, and image distance
 - changing these settings also changes magnification
 - these settings and sensor size determine field of view
 - convex lenses make real images; concave make virtual images
 - lenses perform a 3D perspective transform of object space

Questions?

Depth of field

LESS DEPTH OF FIELD

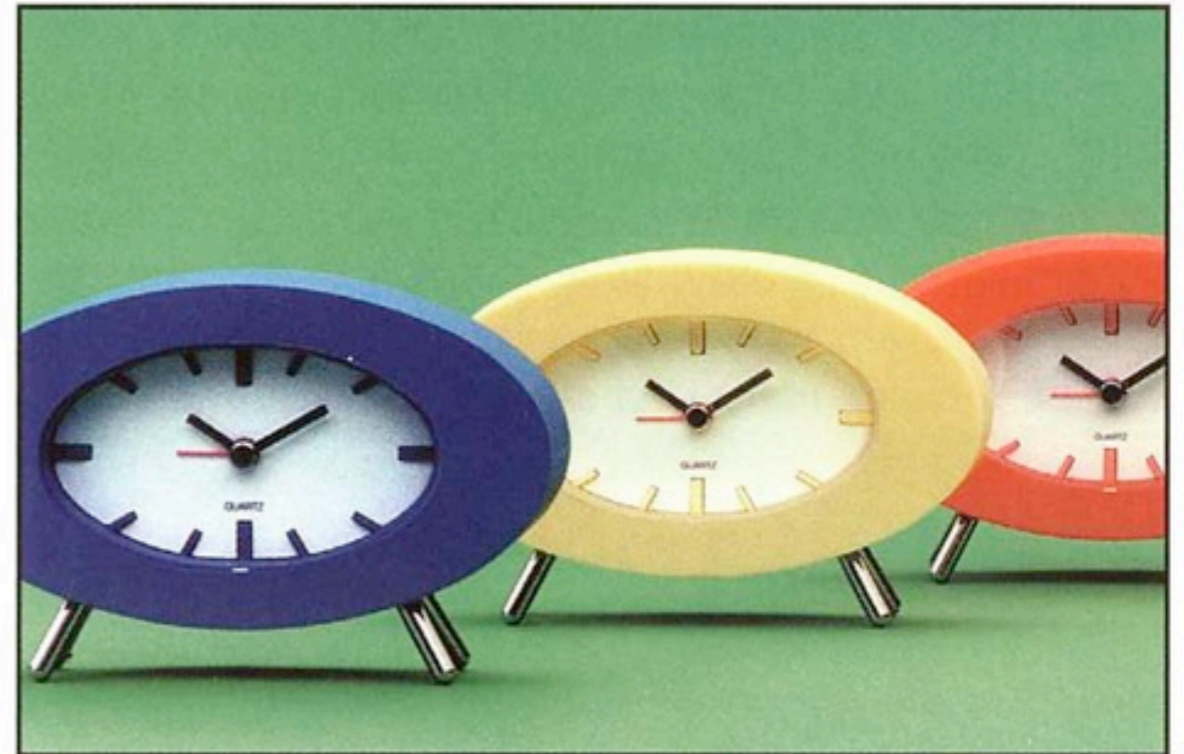


Wider aperture



f/2

MORE DEPTH OF FIELD



Smaller aperture



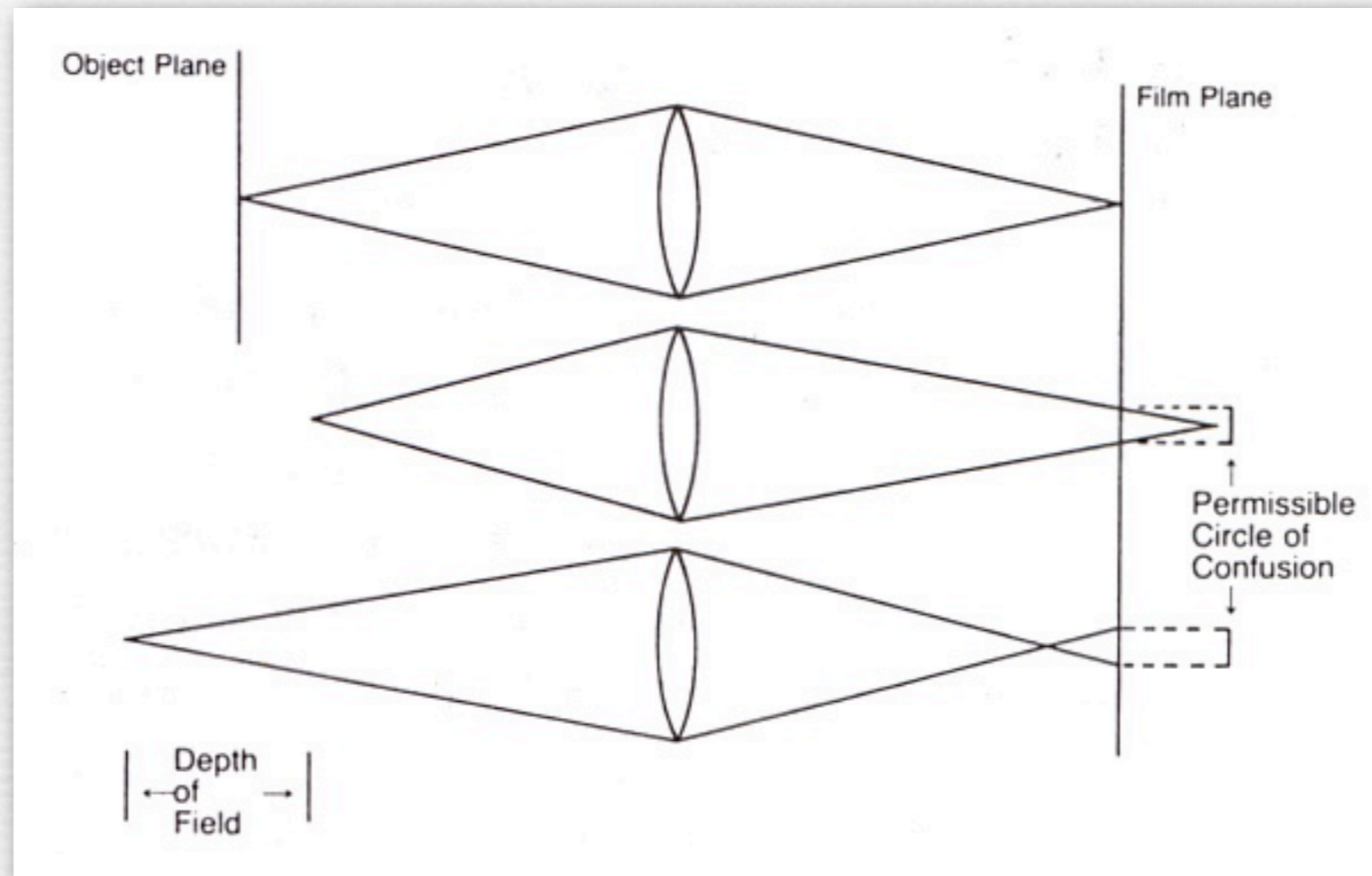
f/16

(London)

$$N = \frac{f}{A}$$

- ◆ lower N means a wider aperture and less depth of field

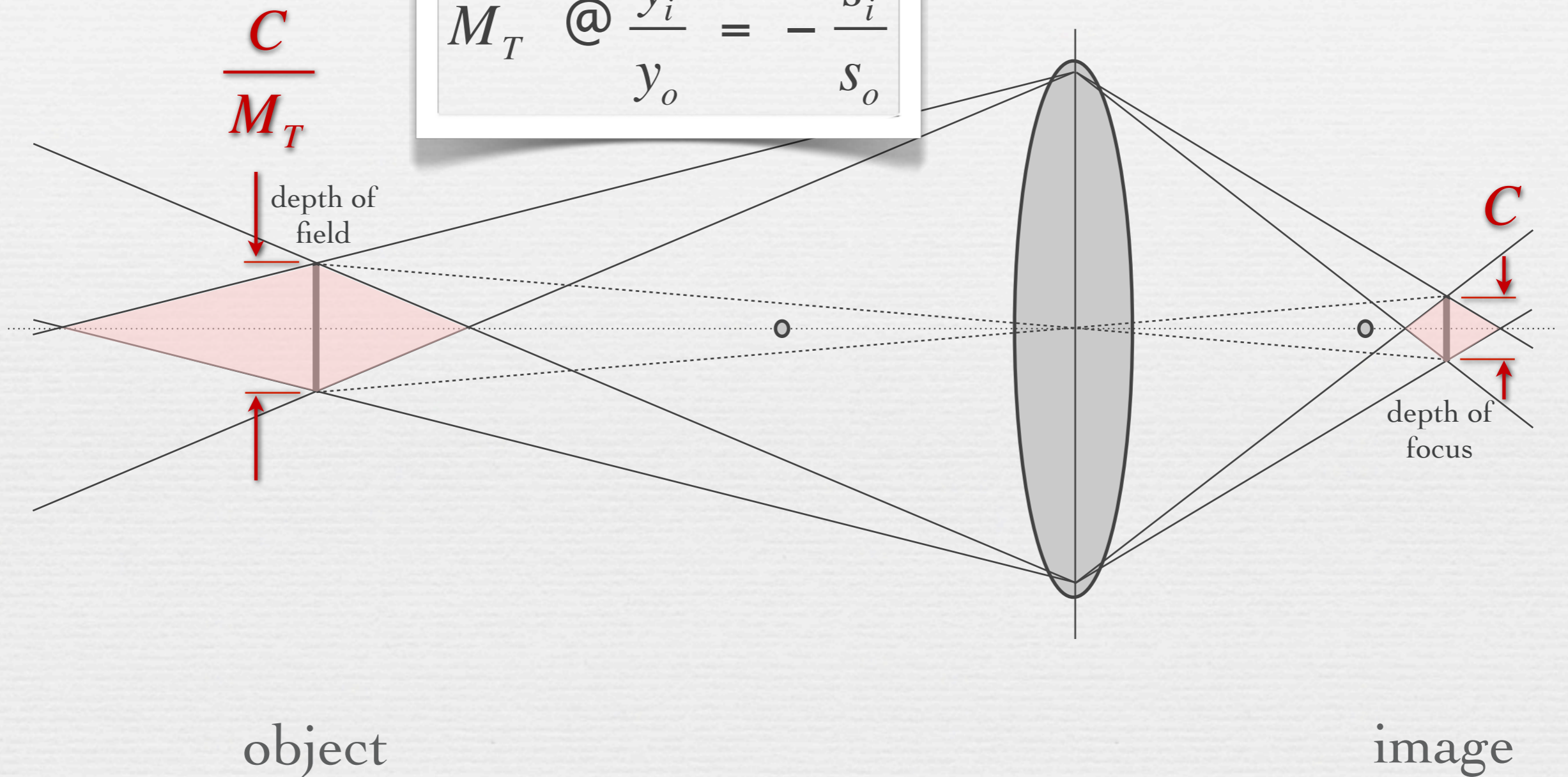
Circle of confusion (C)



- ◆ C depends on sensing medium, reproduction medium, viewing distance, human vision, ...
 - for print from 35mm film, 0.02mm (on negative) is typical
 - for high-end SLR, 6 μ is typical (1 pixel)
 - larger if downsizing for web, or lens is poor

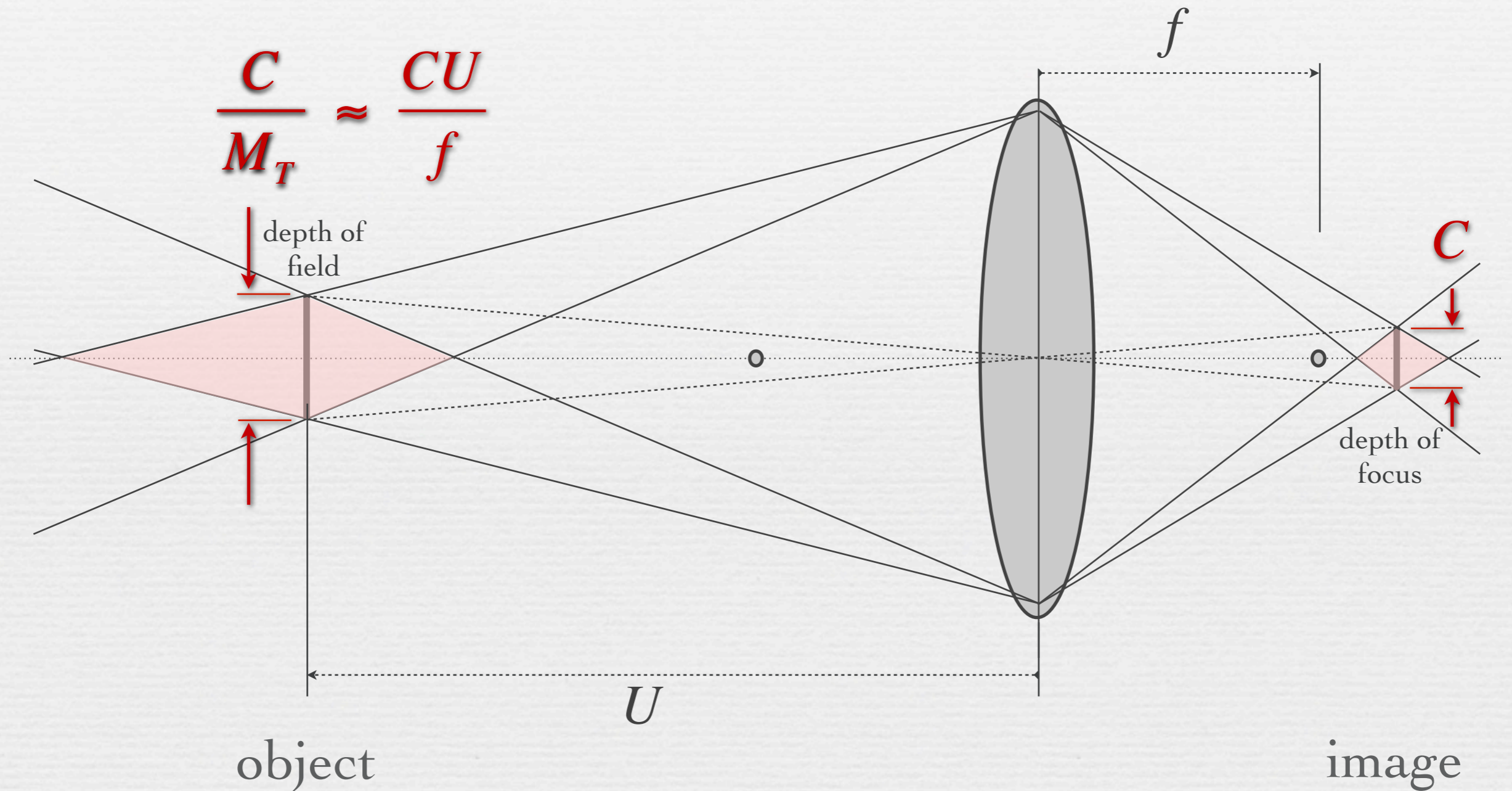
Depth of field formula

$$M_T @ \frac{y_i}{y_o} = - \frac{s_i}{s_o}$$



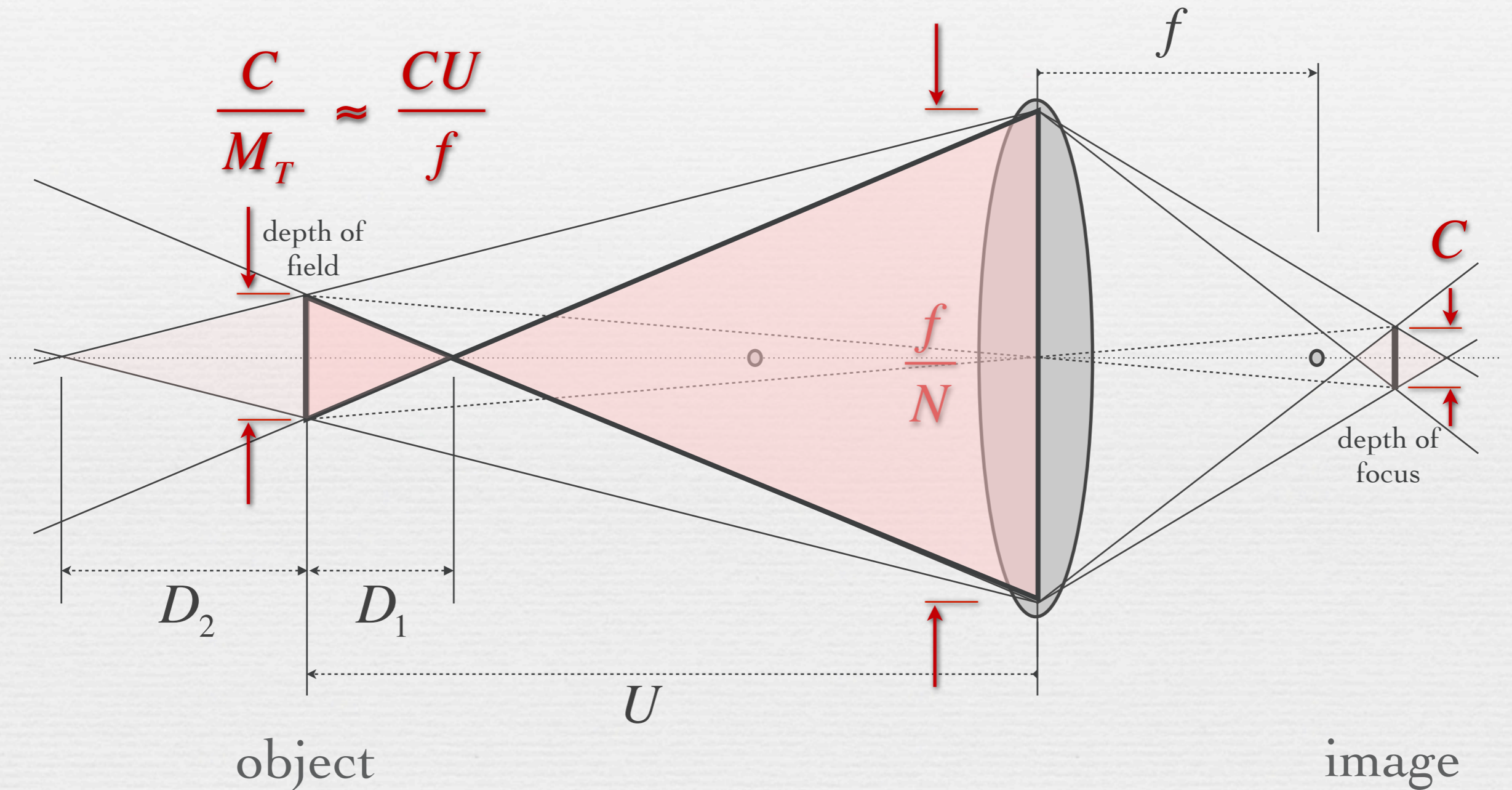
- ◆ DoF is asymmetrical around the in-focus object plane
- ◆ conjugate in object space is typically bigger than C

Depth of field formula



- ◆ DoF is asymmetrical around the in-focus object plane
- ◆ conjugate in object space is typically bigger than C

Depth of field formula



$$\frac{C}{M_T} \approx \frac{CU}{f}$$

$$\frac{D_1 f}{CU} = \frac{U - D_1}{f / N} \dots D_1 = \frac{NCU^2}{f^2 + NCU} \quad D_2 = \frac{NCU^2}{f^2 - NCU}$$


Depth of field formula

$$D_{TOT} = D_1 + D_2 = \frac{2NCU^2 f^2}{f^4 - N^2 C^2 U^2}$$

- ◆ $N^2 C^2 U^2$ can be ignored when conjugate of circle of confusion is small relative to the aperture

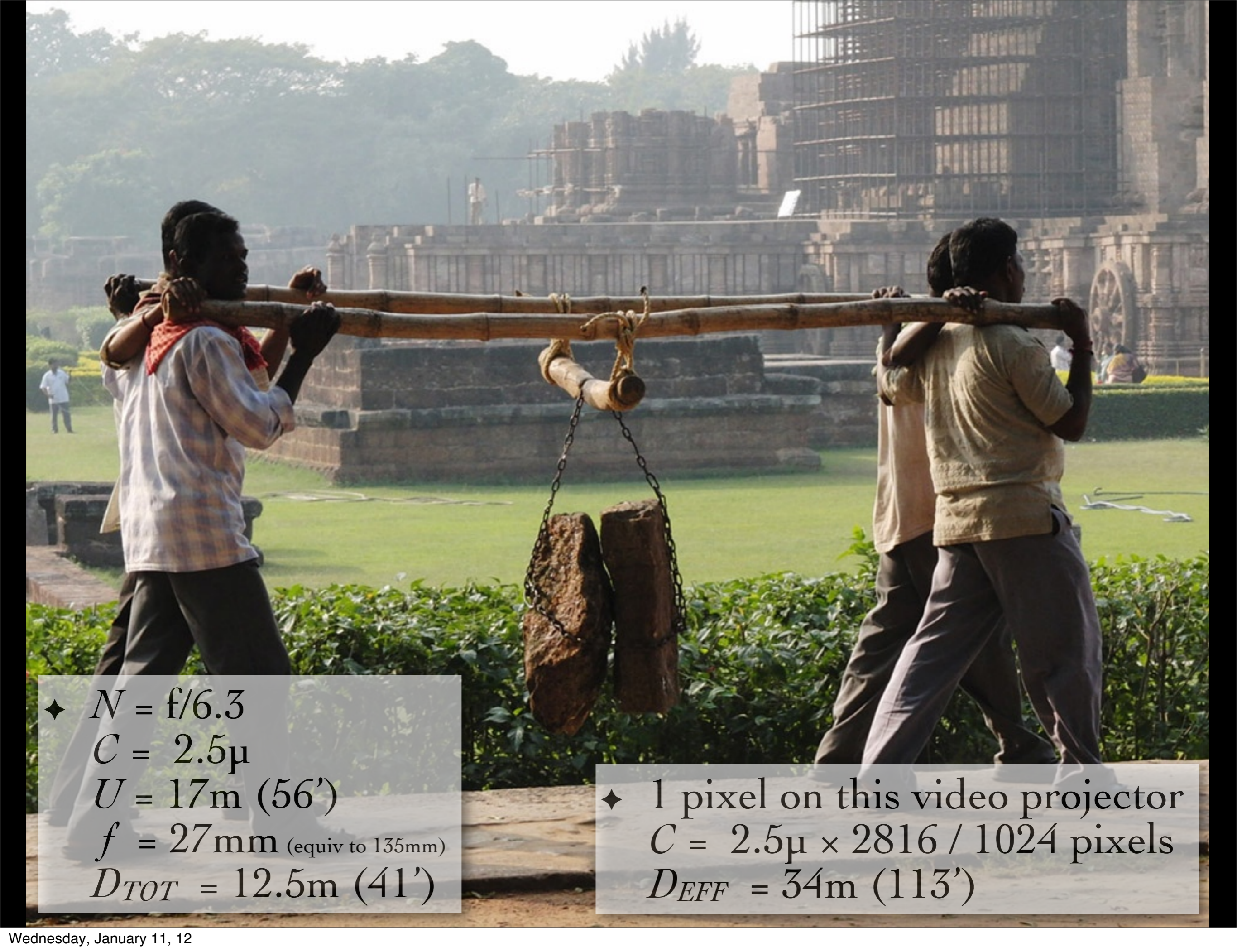
$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- ◆ where
 - N is F-number of lens
 - C is circle of confusion (on image)
 - U is distance to in-focus plane (in object space)
 - f is focal length of lens


$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- ◆ $N = f/4.1$
- $C = 2.5\mu$
- $U = 5.9\text{m (19')}$
- $f = 73\text{mm (equiv to 362mm)}$
- $D_{TOT} = 132\text{mm}$

- ◆ 1 pixel on this video projector
 $C = 2.5\mu \times 2816 / 1024$ pixels
 $D_{EFF} = 363\text{mm}$



◆ $N = f/6.3$
 $C = 2.5\mu$
 $U = 17\text{m (56')}$
 $f = 27\text{mm (equiv to 135mm)}$
 $D_{TOT} = 12.5\text{m (41')}$

◆ 1 pixel on this video projector
 $C = 2.5\mu \times 2816 / 1024$ pixels
 $D_{EFF} = 34\text{m (113')}$



◆ $N = f/5.6$
 $C = 6.4\mu$
 $U = 0.7\text{m}$
 $f = 105\text{mm}$
 $D_{TOT} = 3.2\text{mm}$

◆ 1 pixel on this video projector
 $C = 6.4\mu \times 5616 / 1024$ pixels
 $D_{EFF} = 17.5\text{mm}$



Canon MP-E
65mm 5:1 macro

◆ $N = f/2.8$
 $C = 6.4\mu$
 $U = 78\text{mm}$
 $f = 65\text{mm}$

(use $N' = (1+M_T)N$ at short conjugates ($M_T=5$ here)) = f/16

$D_{TOT} = 0.29\text{mm!}$



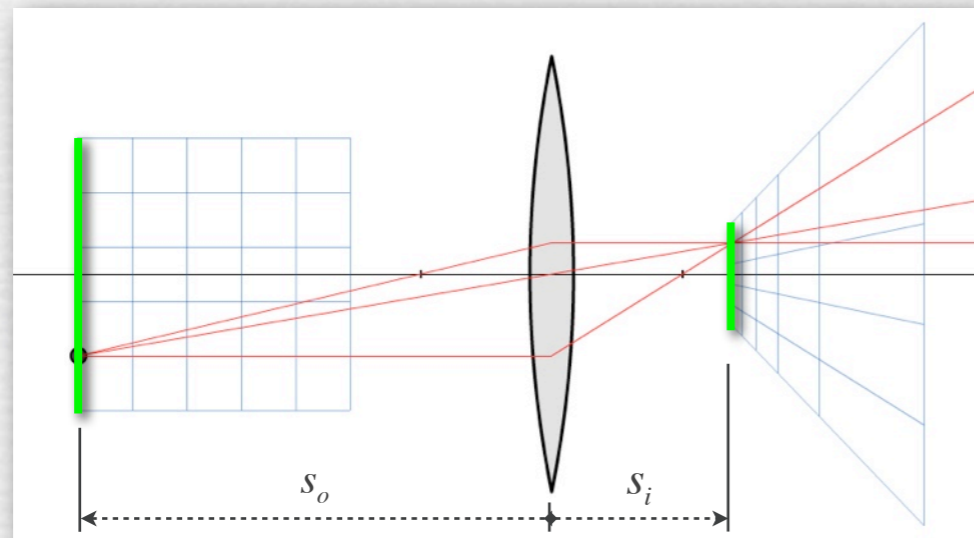
(Mikhail Shlemov)

Sidelight: macro lenses

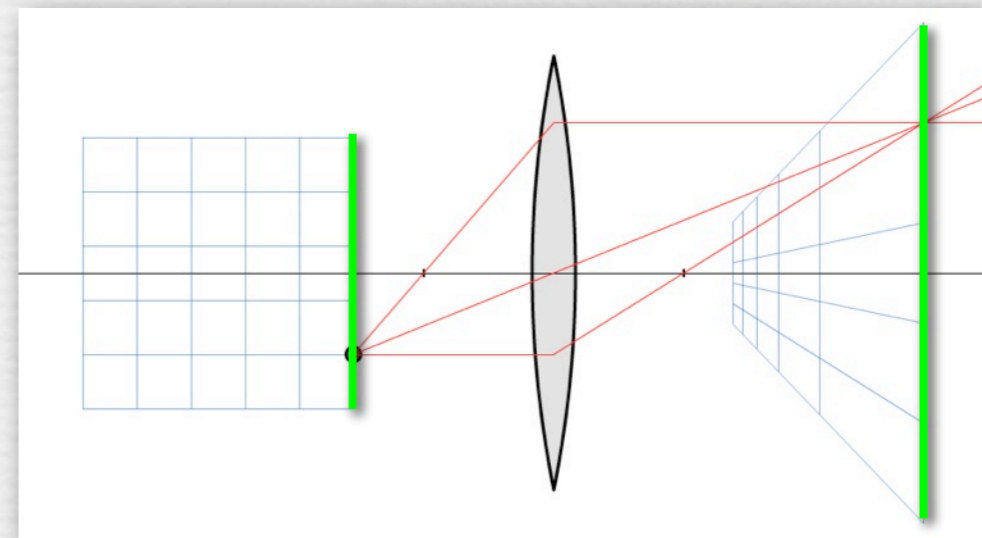
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$



Q. How can the Casio EX-F1 at 73mm and the Canon MP-E 65mm macro, which have similar f 's, have such different focusing distances?



normal



macro

- ◆ A. Because they are built to allow different s_i
 - this changes s_o , which changes magnification M_T @ $-s_i / s_o$
 - macro lenses allow long s_i and they are well corrected for aberrations at short s_o

Hyperfocal distance

- ◆ the back depth of field

$$D_2 = \frac{NCU^2}{f^2 - NCU}$$

- ◆ becomes infinite if

$$U \geq \frac{f^2}{NC} \text{ @ } H$$

- ◆ In that case, the front depth of field becomes

$$D_1 = \frac{H}{2}$$

- ◆ so if I had focused at 32m, everything from 16m to infinity would be in focus on a video projector, including the men at 17m



- ◆ $N = f/6.3$
 $C = 2.5\mu \times 2816 / 1024$ pixels
 $U = 17\text{m}$ (56')
 $f = 27\text{mm}$ (equiv to 135mm)
 $D_{TOT} = 34\text{m}$ on video projector
 $H = 32\text{m}$ (106')

(FLASH DEMO)

<http://graphics.stanford.edu/courses/cs178/applets/dof.html>

Q. Does sensor size affect DoF?

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- ◆ as sensor shrinks, lens focal length f typically shrinks to maintain a comparable field of view
- ◆ as sensor shrinks, pixel size C typically shrinks to maintain a comparable number of pixels in the image
- ◆ thus, depth of field D_{TOT} increases linearly with decreasing sensor size
- ◆ this is why amateur cinematographers are drawn to SLRs
 - their chips are larger than even pro-level video camera chips
 - so they provide unprecedented control over depth of field

DoF and the dolly-zoom

- ◆ if we zoom in (change f) and stand further back (change U) by the same factor

$$D_{TOT} \approx \frac{2NC \boxed{U^2}}{\boxed{f^2}}$$

- ◆ the depth of field at the subject stays the same!
 - useful for macro when you can't get close enough



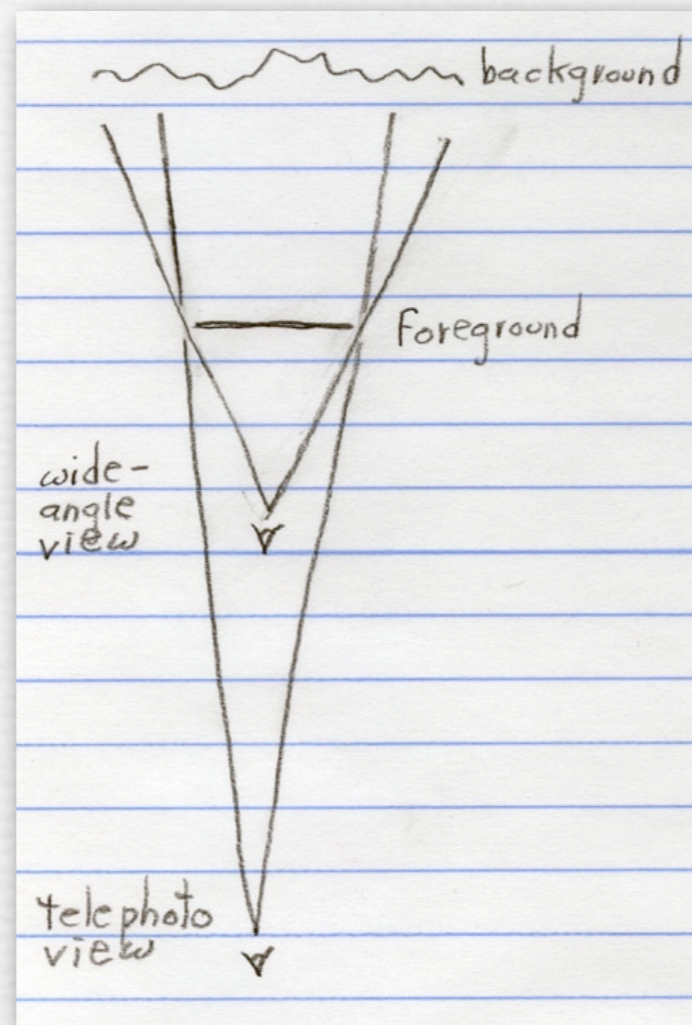
50mm f/4.8



200mm f/4.8,
moved back 4× from subject

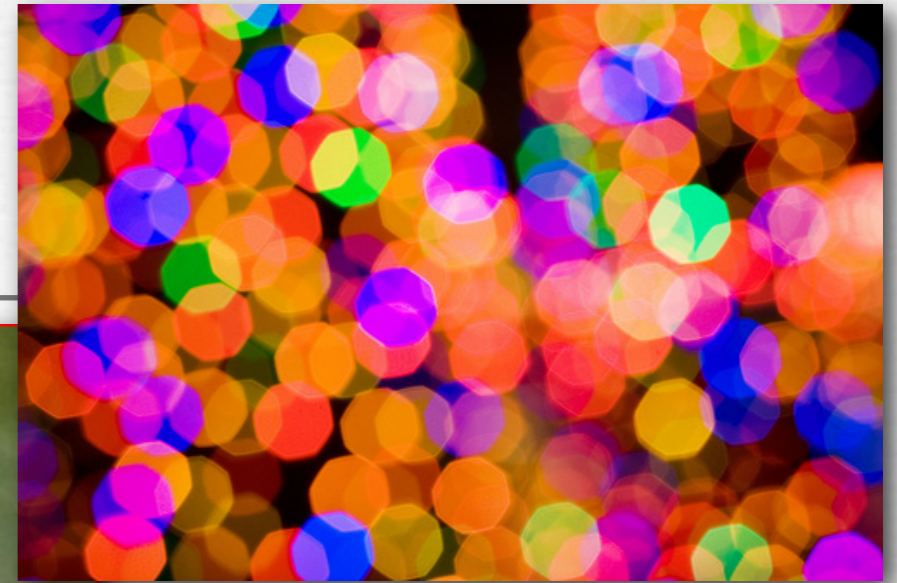
(juzaphoto.com)

Macro photography using a telephoto lens (contents of whiteboard)



- ◆ changing from a wide-angle lens to a telephoto lens and stepping back, you can make a foreground object appear the same size in both lenses
- ◆ and both lenses will have the same depth of field on that object
- ◆ but the telephoto sees a smaller part of the background (which it blows up to fill the field of view), so the background will appear blurrier

Parting thoughts on DoF: the zen of *bokeh*



Canon 85mm
prime f/1.8 lens

- ◆ the appearance of small out-of-focus features in a photograph with shallow depth of field
 - determined by the shape of the aperture
 - people get religious about it
 - but not every picture with shallow DoF has evident bokeh...

Games with bokeh



- ◆ picture by Alice Che (CS 178, 2010)
 - heart-shaped mask in front of lens
 - subject was Christmas lights
 - photograph was misfocused and under-exposed

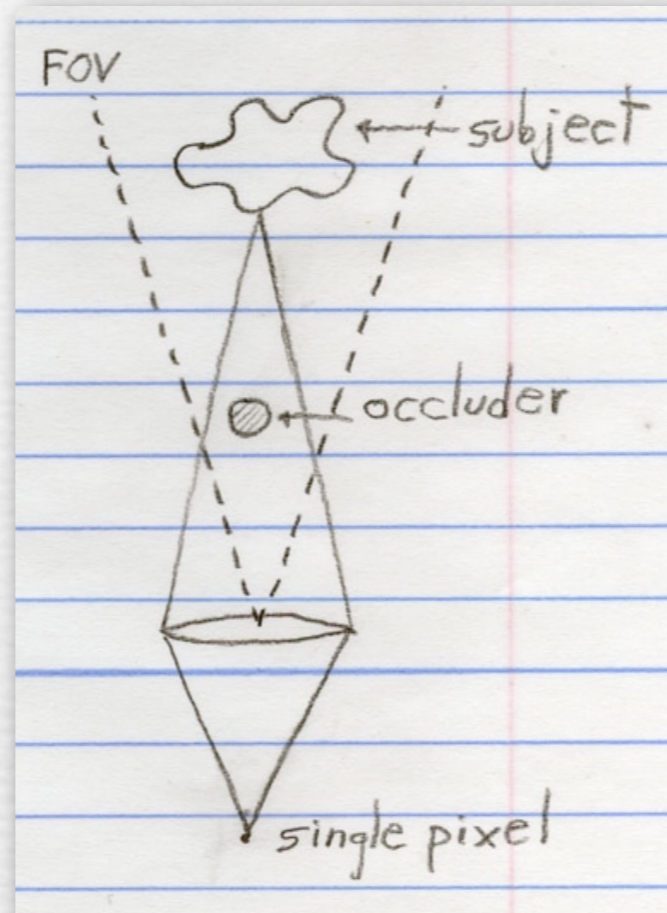
Parting thoughts on DoF: seeing through occlusions



(Fredo Durand)

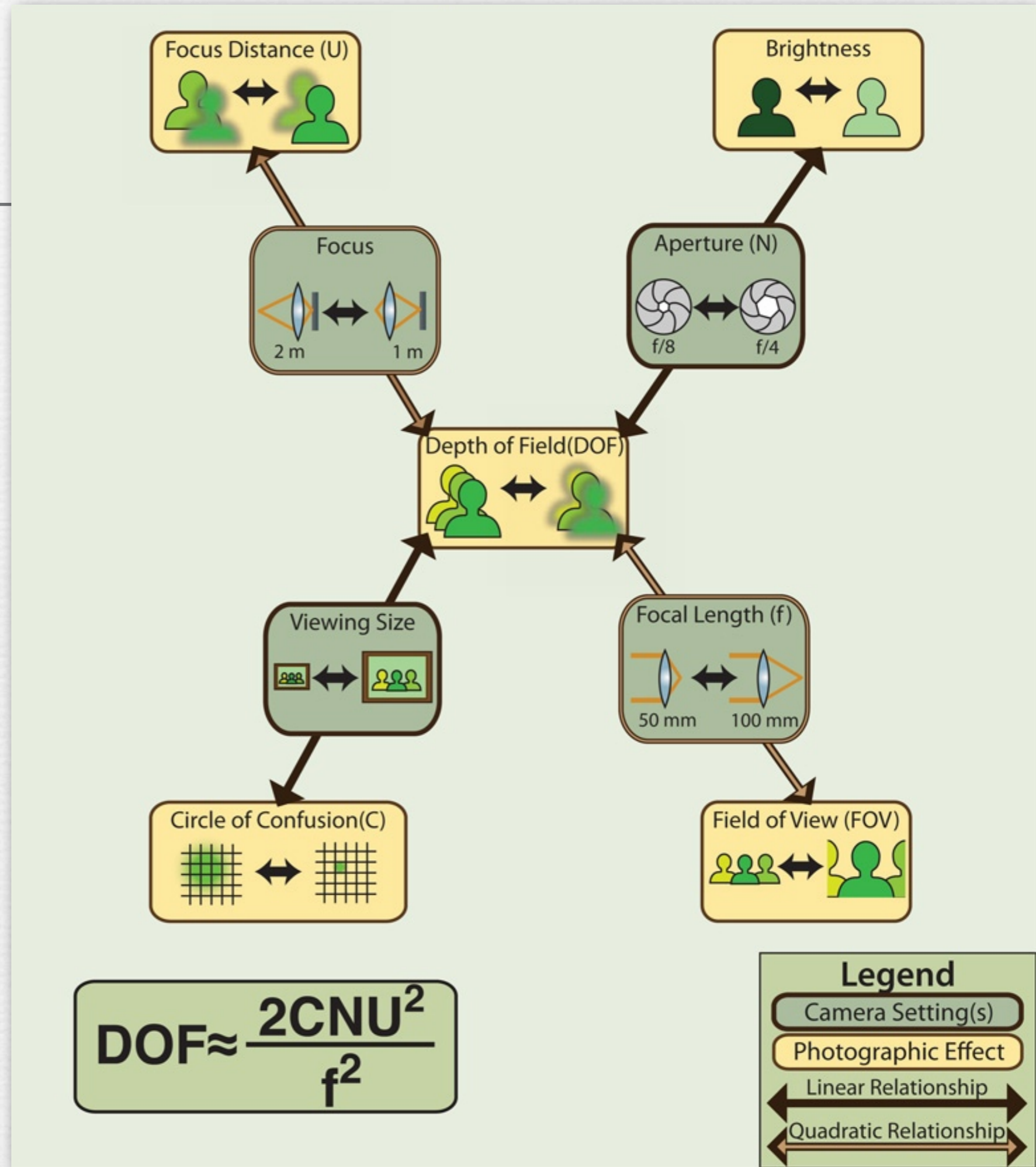
- ◆ depth of field is not a convolution of the image
 - i.e. not the same as blurring in Photoshop
 - DoF lets you eliminate occlusions, like a chain-link fence

Seeing through occlusions using a large aperture (contents of whiteboard)



- ◆ for a pixel focused on the subject, some of its rays will strike the occluder, but some will pass to the side of it, if the occluder is small enough
- ◆ the pixel will then be a mixture of the colors of the subject and occluder
- ◆ thus, the occluder reduces the contrast of your image of the subject, but it doesn't actually block your view of it

Tradeoffs affecting depth of field



Recap

- ◆ depth of field (D_{TOT}) is governed by circle of confusion (C), aperture size (N), subject distance (U), and focal length (f)

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- depth of field is linear in some terms and quadratic in others
 - if you focus at the hyperfocal distance $H = f^2 / NC$, everything from $H / 2$ to infinity will be in focus
 - depth of field increases linearly with decreasing sensor size
- ◆ useful sidelights
 - bokeh refers to the appearance of small out-of-focus features
 - you can take macro photographs using a telephoto lens
 - depth of field blur is not the same as blurring an image

Questions?

Optics II: practical photographic lenses

CS 478, Winter 2012



Slides courtesy of:

Marc Levoy
Computer Science Department
Stanford University

Outline

- ◆ why study lenses?
 - ◆ thin lenses
 - graphical constructions, algebraic formulae
 - ◆ thick lenses
 - center of perspective, 3D perspective transformations
 - ◆ depth of field
-
- ◆ aberrations & distortion
 - ◆ vignetting, glare, and other lens artifacts
 - ◆ diffraction and lens quality
 - ◆ special lenses
 - telephoto, zoom

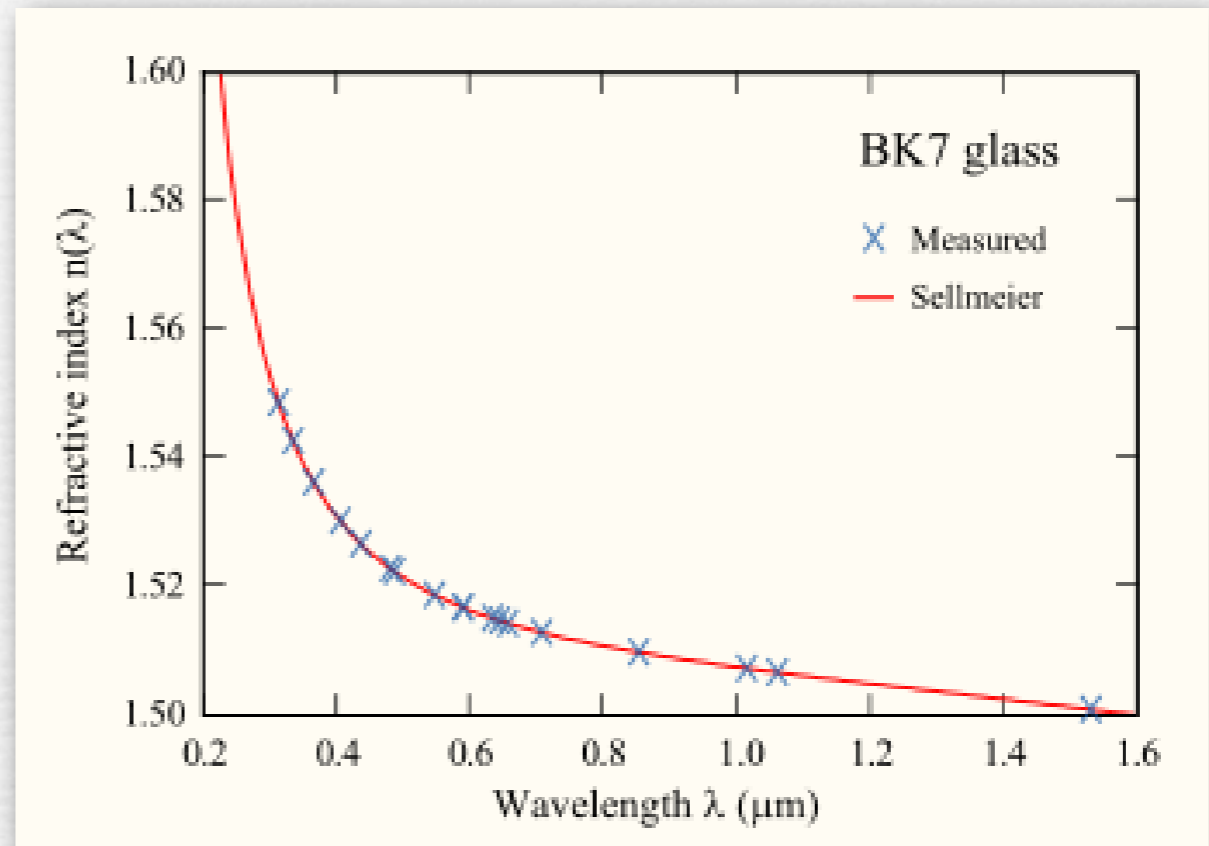
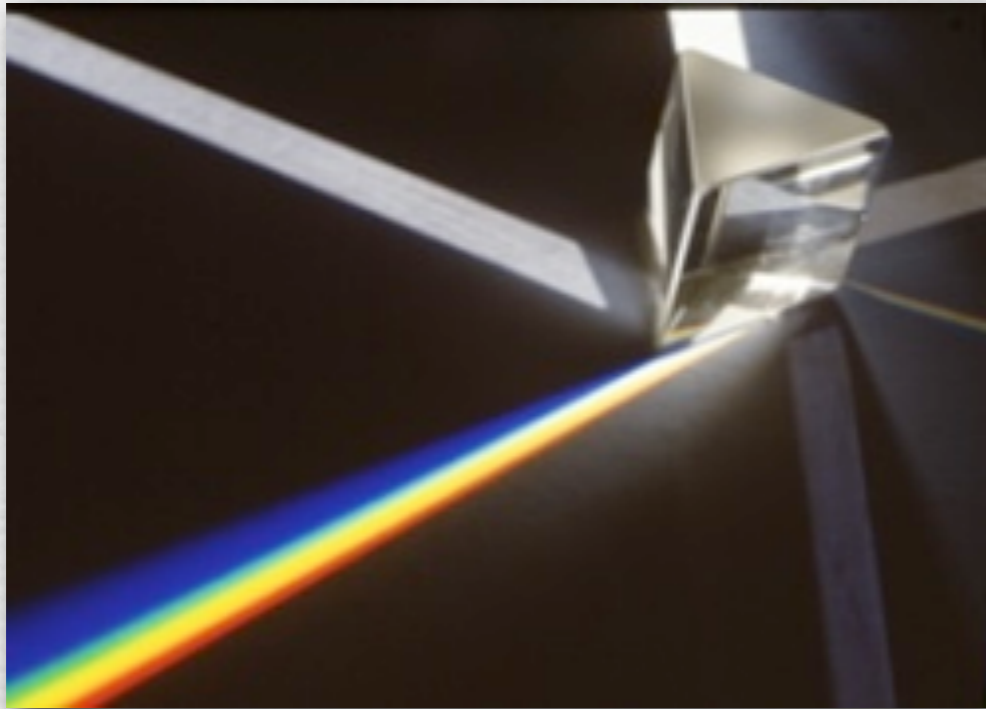
Lens aberrations

- ◆ chromatic aberrations
- ◆ Seidel aberrations, a.k.a. 3rd order aberrations
 - arise because of error in our 1st order approximation

$$\sin \phi \approx \phi \left(-\frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \dots \right)$$

- spherical aberration
- oblique aberrations
- field curvature
- distortion

Dispersion

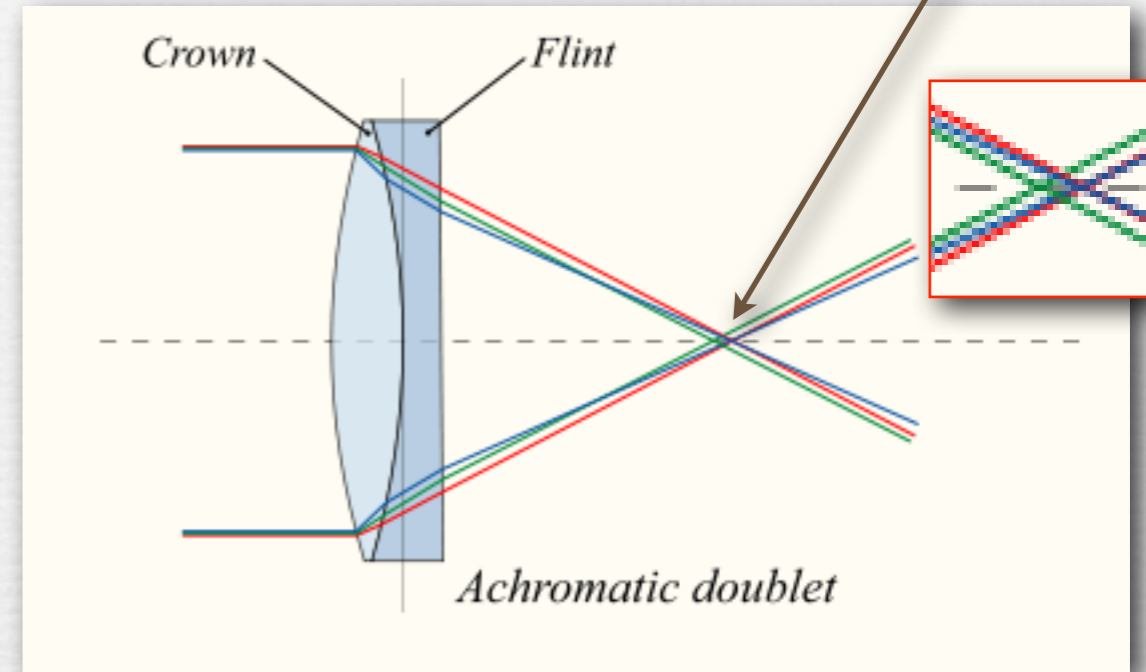
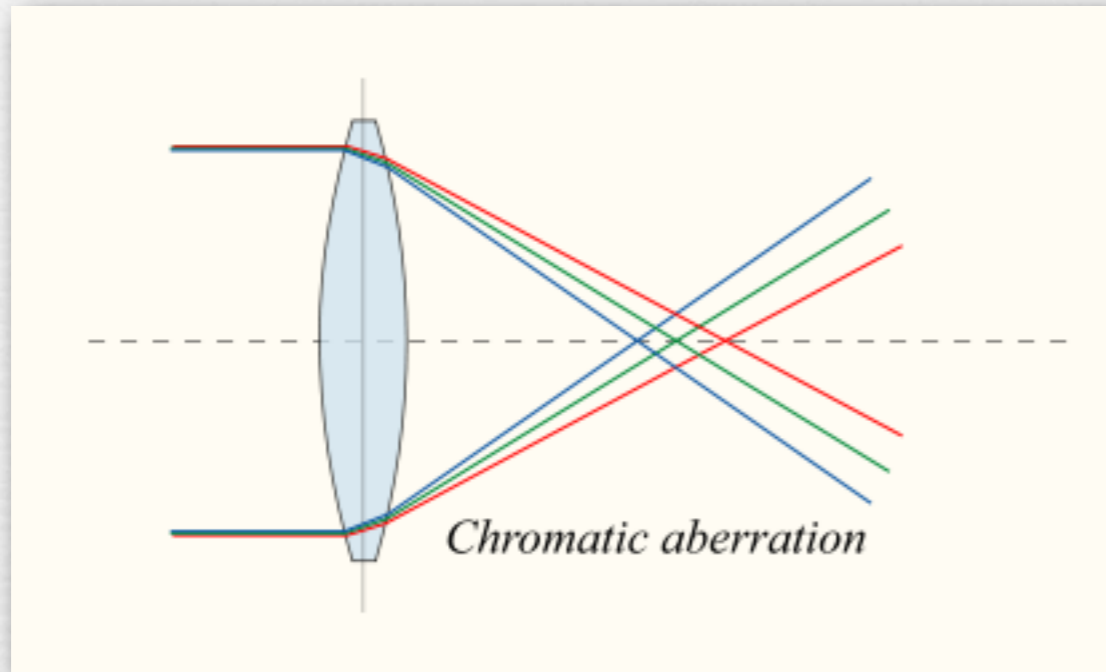


(wikipedia)

- ◆ index of refraction varies with wavelength
 - higher dispersion means more variation
 - amount of variation depends on material
 - index is typically higher for blue than red
 - so blue light bends more

Chromatic aberration

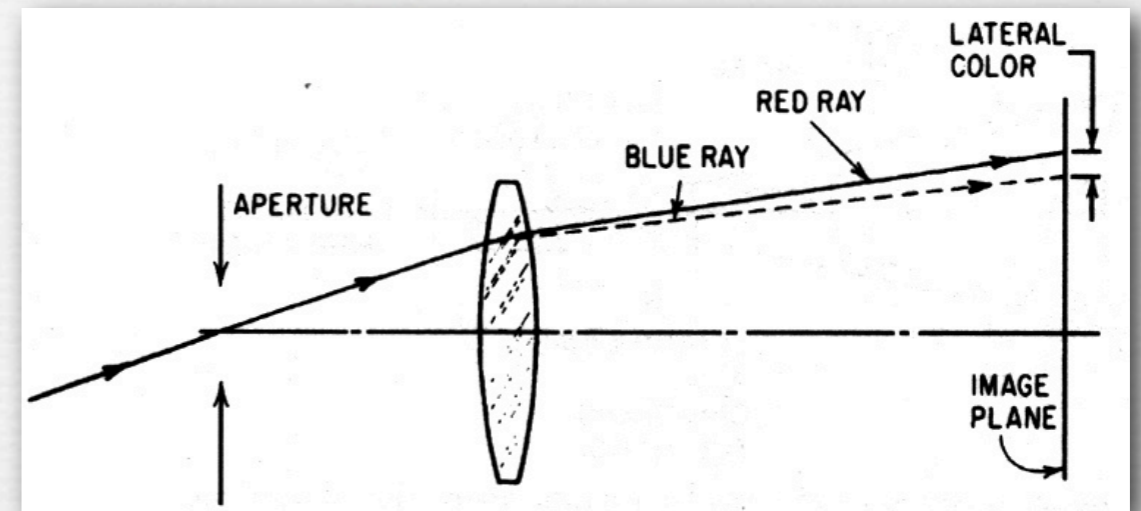
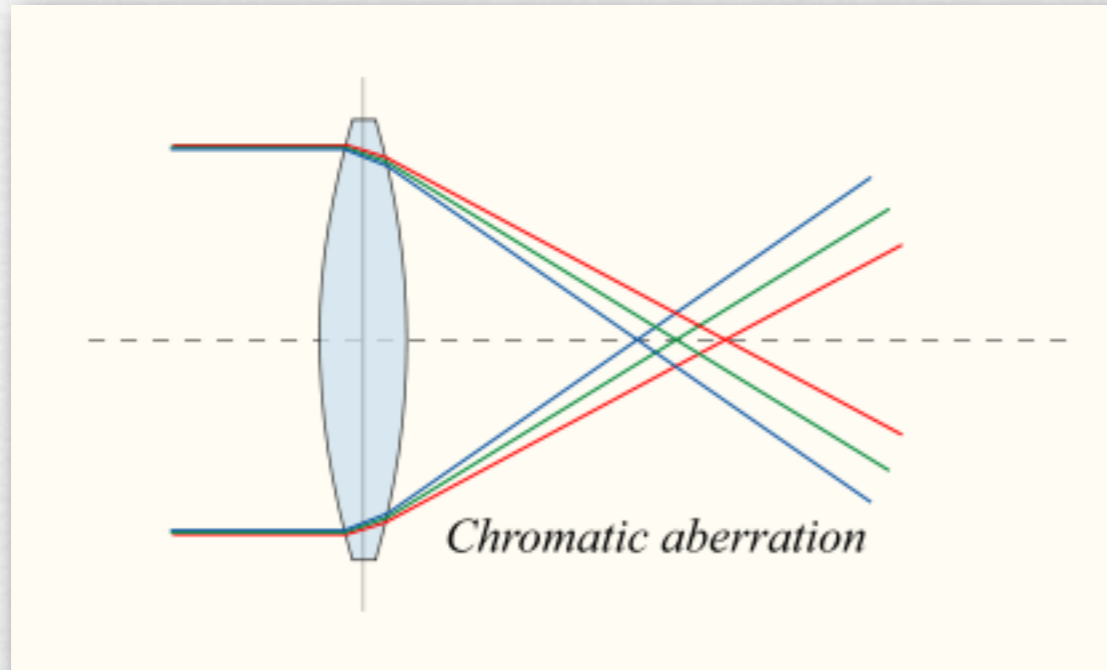
red and blue have
the same focal length



(wikipedia)

- ◆ dispersion causes focal length to vary with wavelength
 - for convex lens, blue focal length is shorter
- ◆ correct using *achromatic doublet*
 - strong positive lens + weak negative lens = weak positive compound lens
 - by adjusting dispersions, can correct at two wavelengths

The chromatic aberrations



- ◆ change in focus with wavelength
 - called *longitudinal (axial) chromatic aberration*
 - appears everywhere in the image
- ◆ if blue image is closer to lens, it will also be smaller
 - called *lateral (transverse) chromatic aberration*
 - only appears at edges of images, not in the center
- ◆ can reduce longitudinal by closing down the aperture

Examples

● correctable
in software

● not

(wikipedia)



lateral

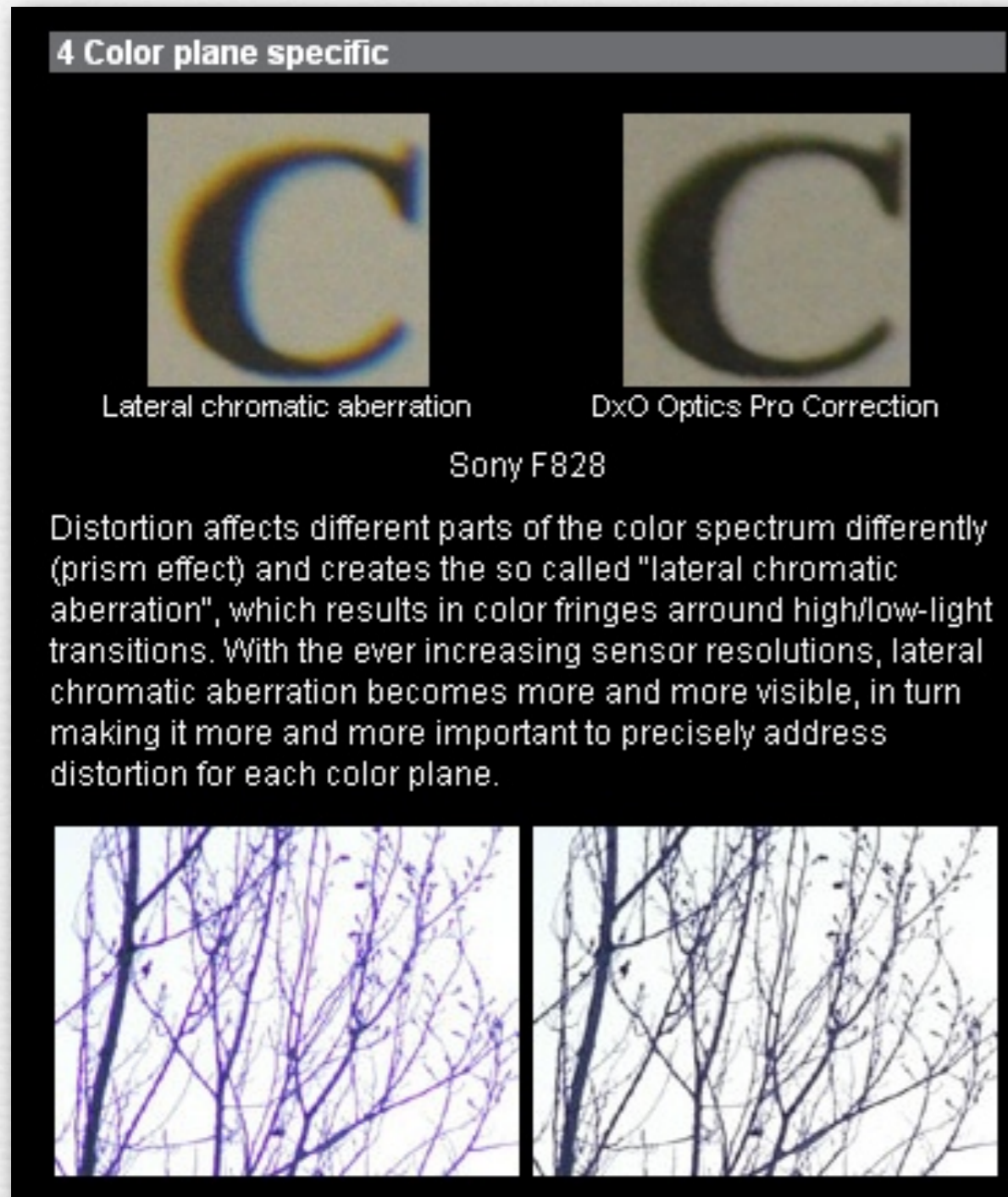
(toothwalker.org)



longitudinal

- ◆ other possible causes
 - demosiacing algorithm
 - per-pixel microlenses
 - lens flare

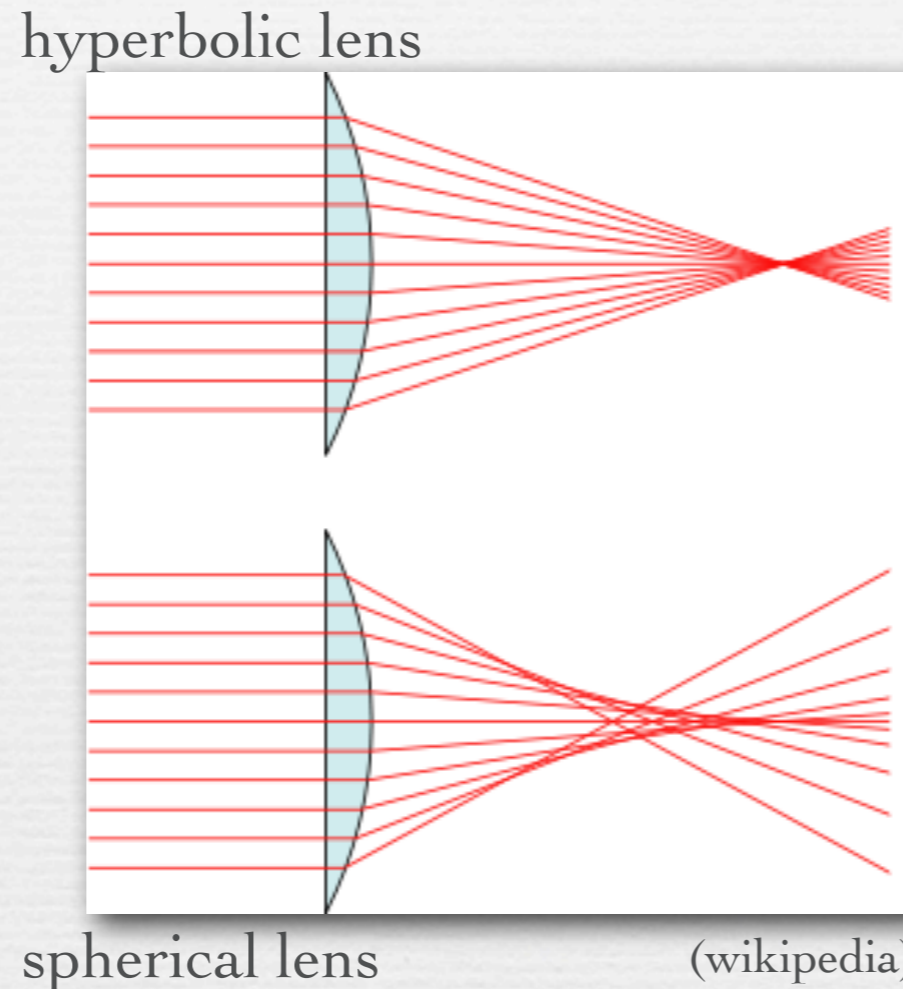
Software correction of lateral chromatic aberration



- ◆ Panasonic GF1 corrects for chromatic aberration in the camera (or in Adobe Camera Raw)
 - need focal length of lens, and focus setting

Q. Why don't humans see chromatic aberration?

Spherical aberration



- ◆ focus varies with ray height (distance from optical axis)
- ◆ can reduce by stopping down the aperture
- ◆ can correct using an aspherical lens
- ◆ can correct for this and chromatic aberration by combining with a concave lens of a different index

Examples



(Canon)

sharp

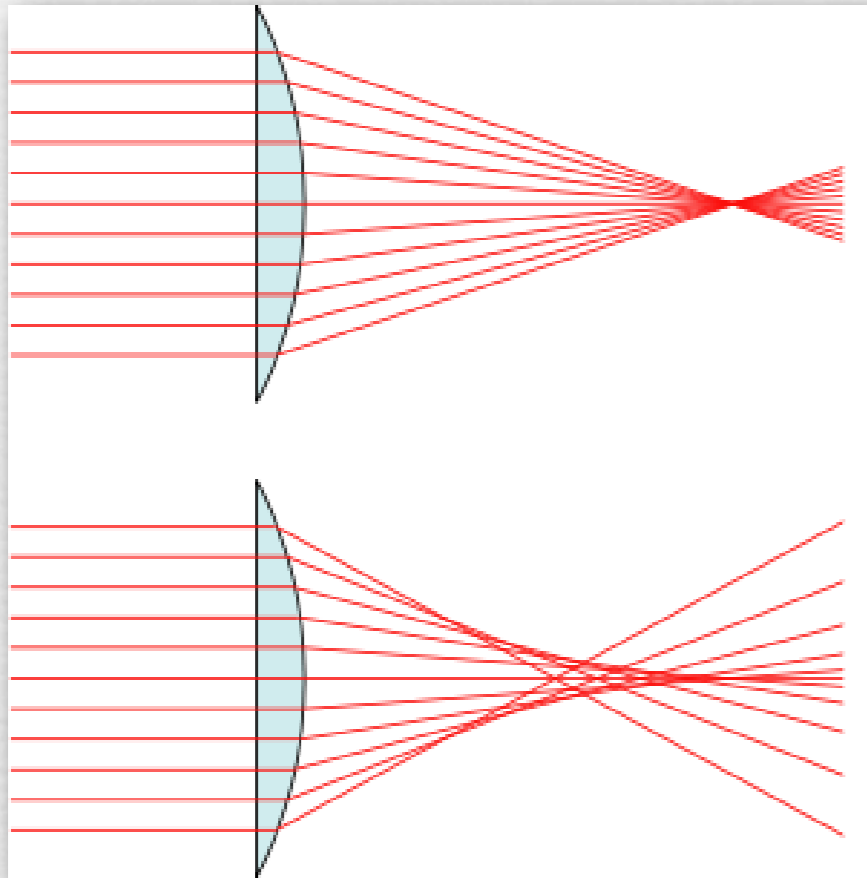


soft focus

Canon 135mm f/2.8 soft focus lens

Focus shift

(diglloyd.com)



(wikipedia)

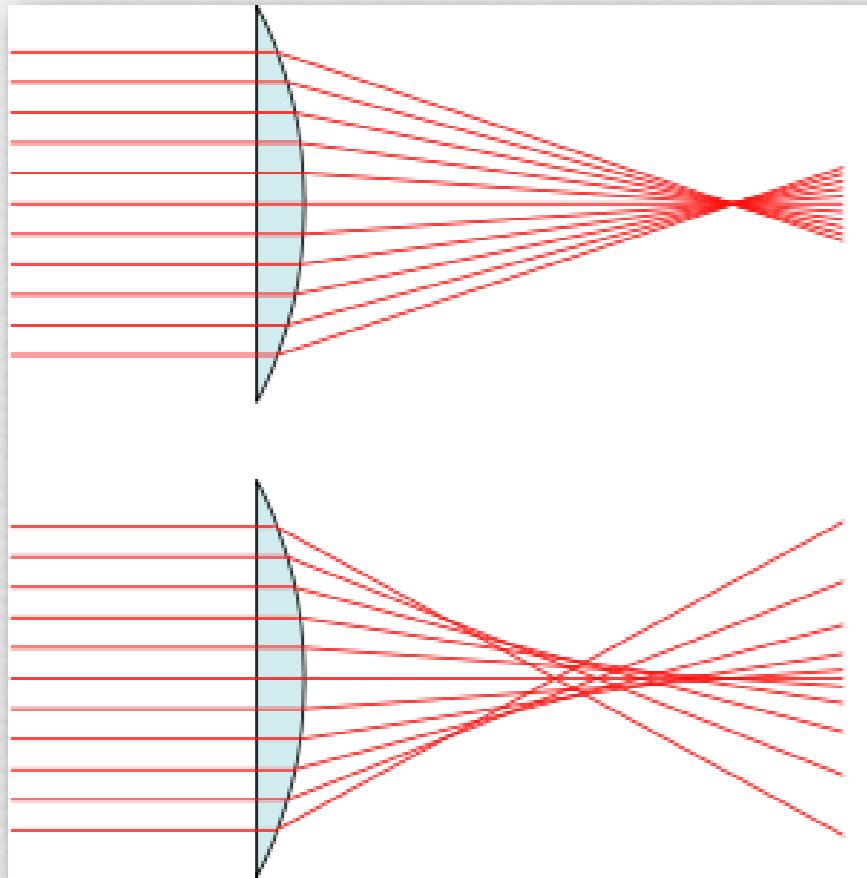


focused at $f/1.2$

◆ Canon 50mm $f/1.2$ L

Focus shift

(diglloyd.com)



(wikipedia)



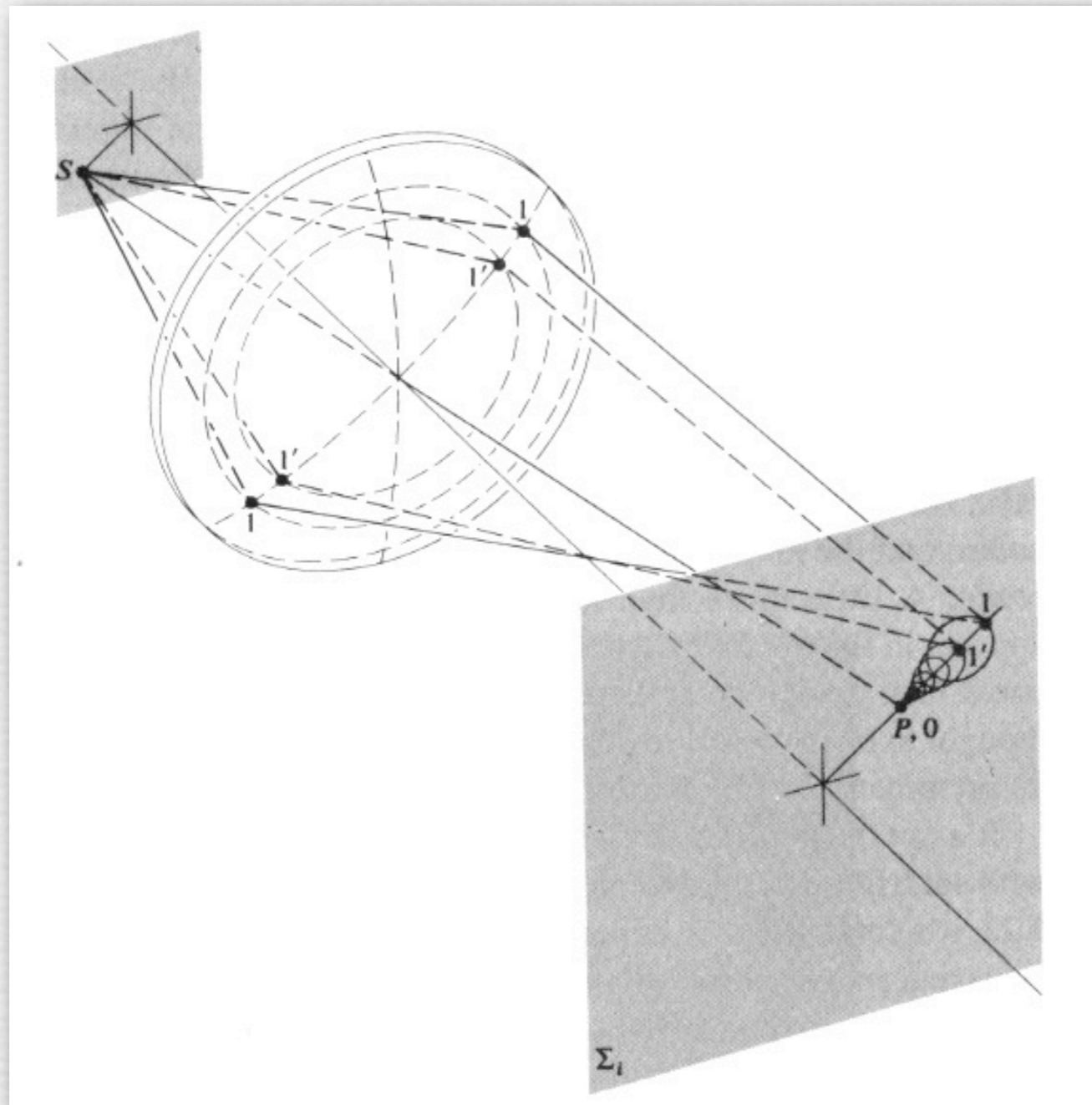
shot at f/1.8

- ◆ Canon 50mm f/1.2 L
- ◆ narrowing the aperture pushed the focus deeper

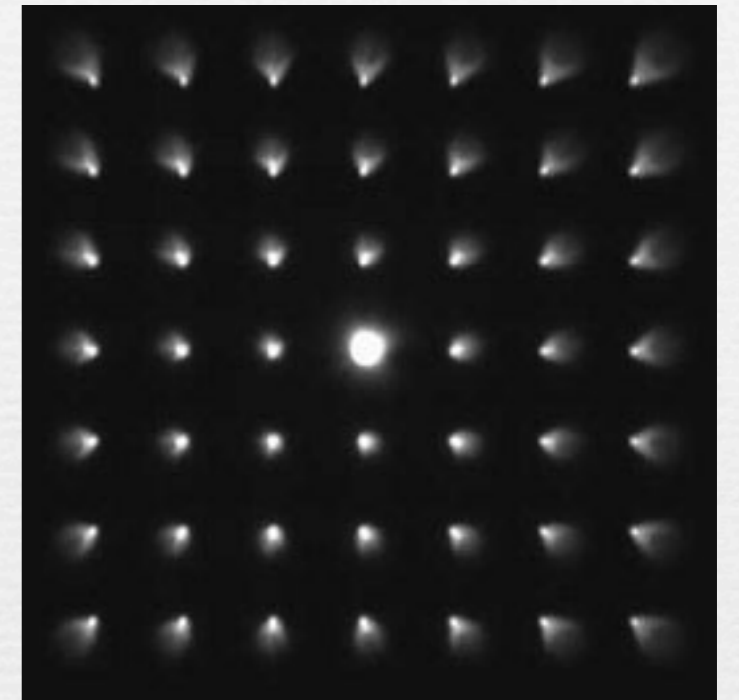
Oblique aberrations

- ◆ lateral chromatic aberrations do not appear in center of field
 - they get worse with increasing distance from the optical axis
 - cannot reduce by closing down the aperture
- ◆ longitudinal chromatic & spherical aberrations occur everywhere in the field of view
 - on and off the optical axis
 - can reduce by closing down the aperture
- ◆ oblique aberrations do not appear in center of field
 - they get worse with increasing distance from the optical axis
 - can reduce by closing down the aperture
 - coma and astigmatism

Coma



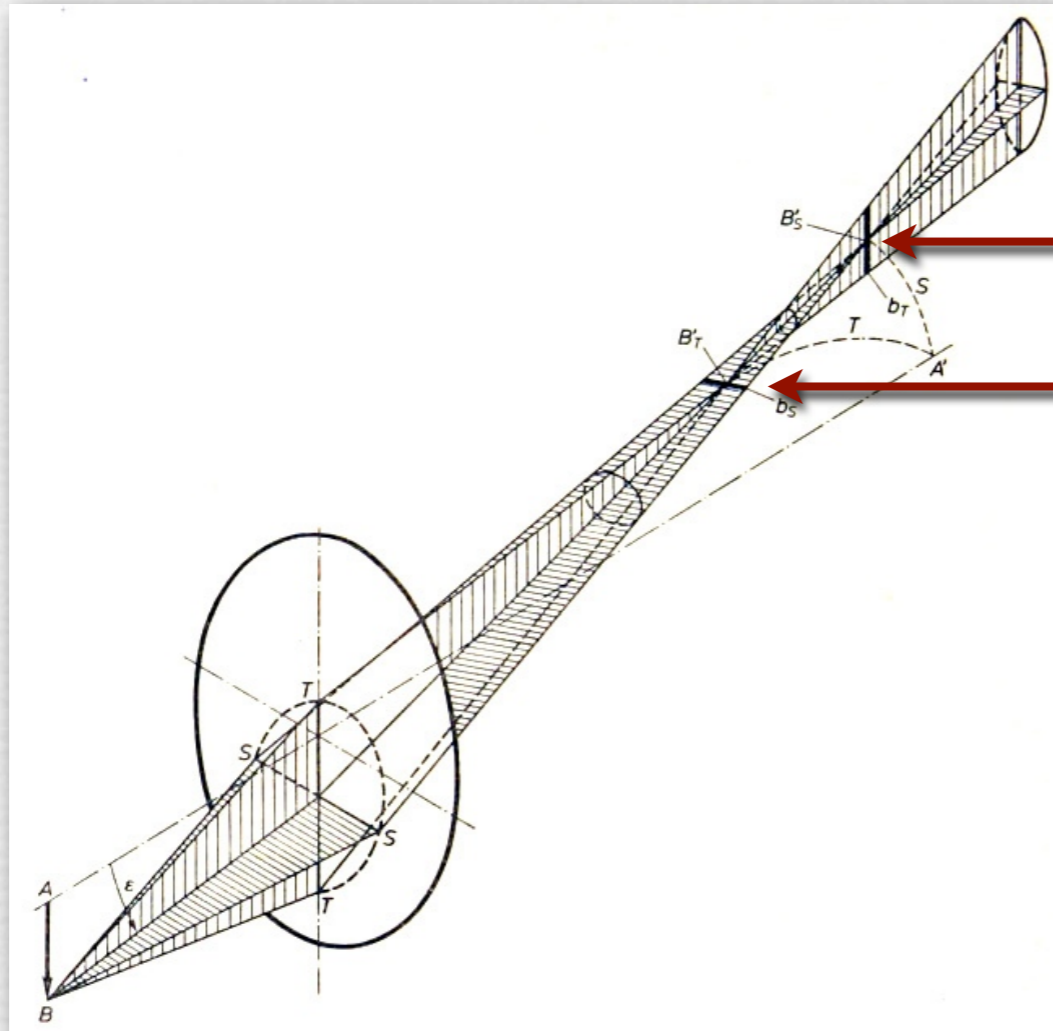
(Hecht)



(ryokosha.com)

- ◆ magnification varies with ray height (distance from optical axis)

Astigmatism

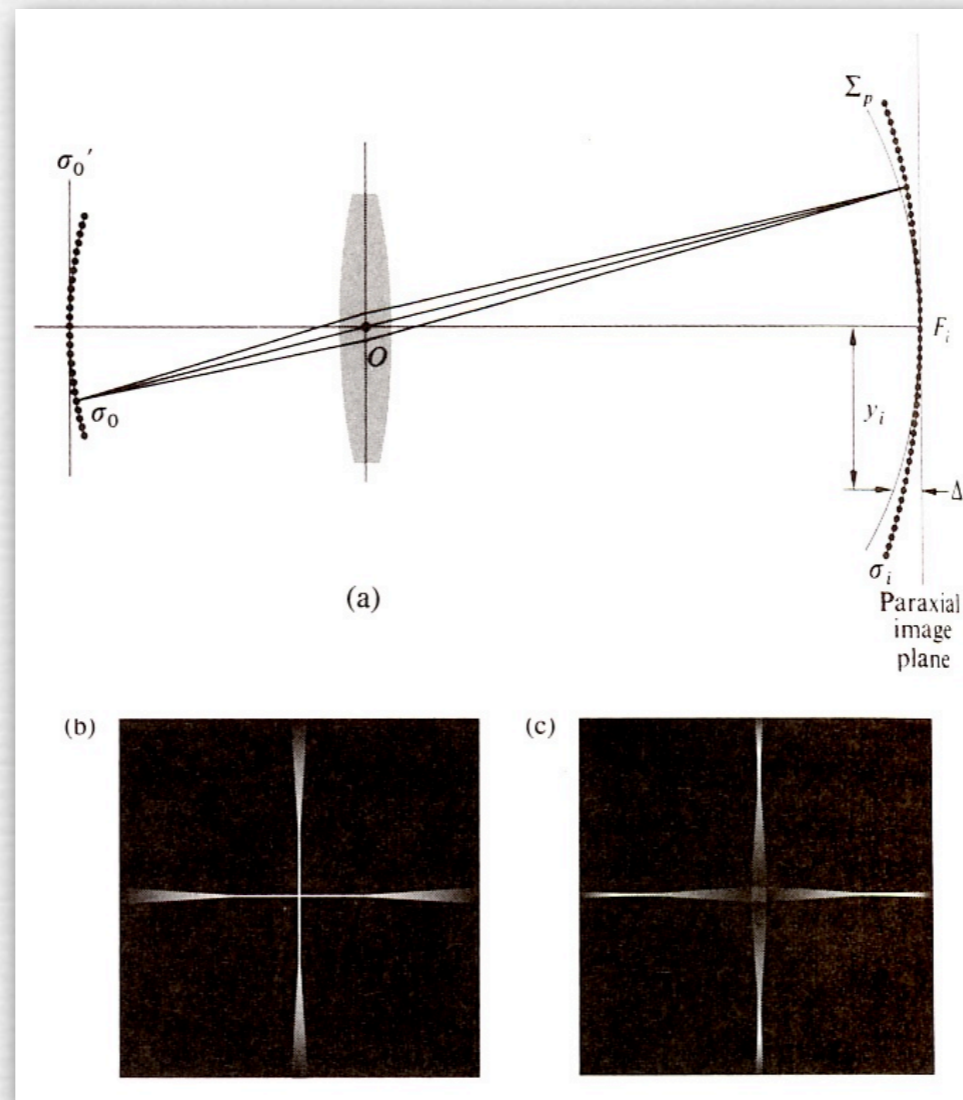


focus of sagittal rays
focus of tangential rays

(Pluta)

- ◆ tangential and sagittal rays focus at different depths

Field curvature

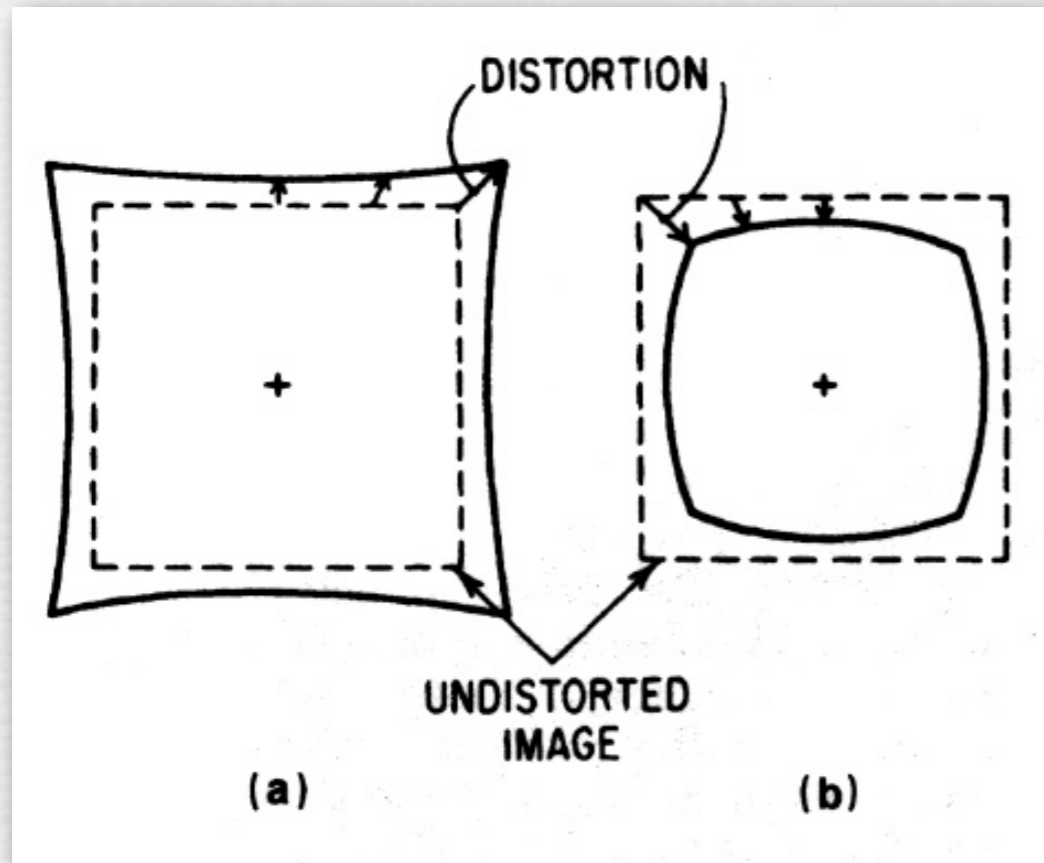


- ◆ spherical lenses focus a curved surface in object space onto a curved surface in image space
- ◆ so a plane in object space cannot be everywhere in focus when imaged by a planar sensor

Distortion

- correctable in software

(Smith)



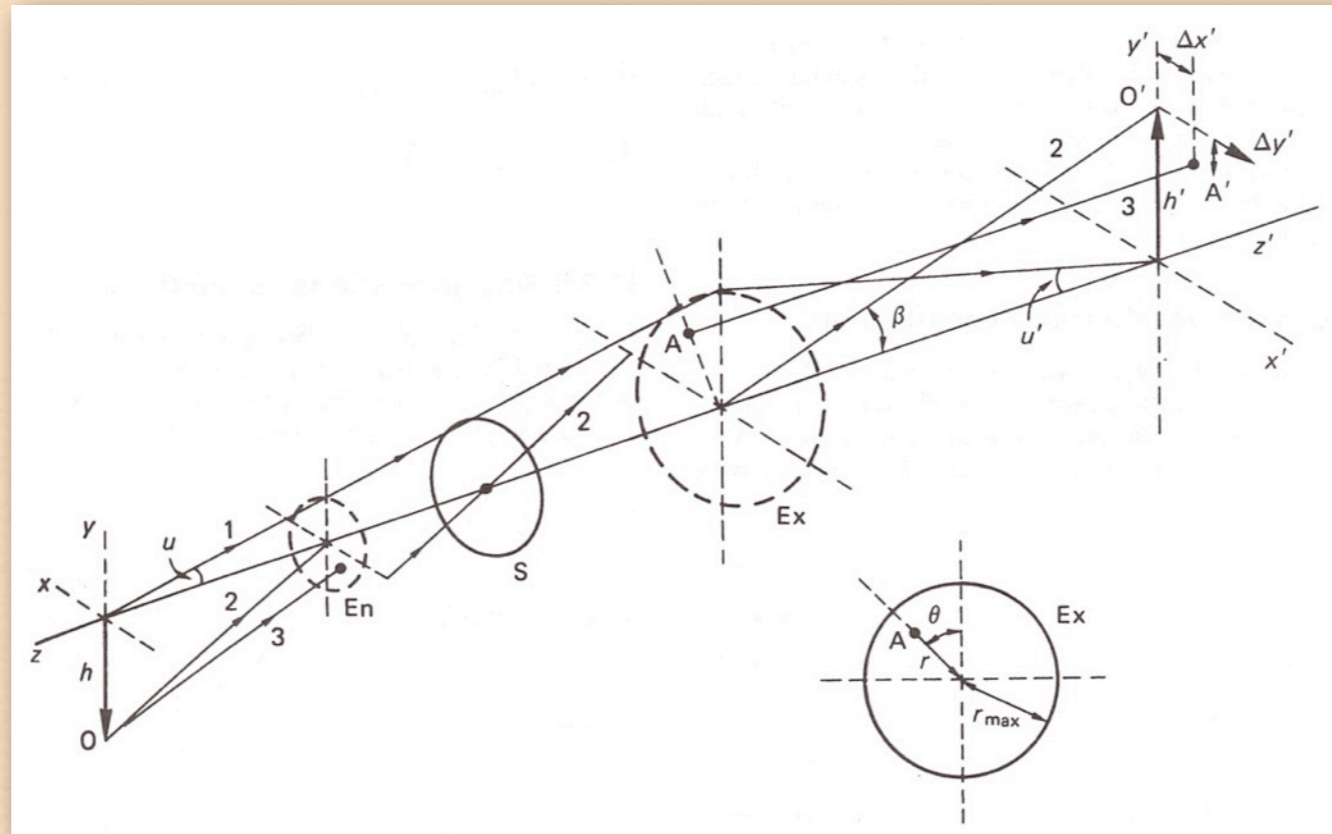
(Kingslake)



pincushion distortion

- ◆ change in magnification with image position
 - (a) pincushion
 - (b) barrel
- ◆ closing down the aperture does not improve this

Algebraic formulation of monochromatic lens aberrations



(Smith)

- ◆ spherical aberration $a_s r^4$
- ◆ coma $a_c h' r^3 \cos \theta$
- ◆ astigmatism $a_a h'^2 r^2 \cos^2 \theta$
- ◆ field curvature $a_d h'^2 r^2$
- ◆ distortion $a_t h'^3 r \cos \theta$

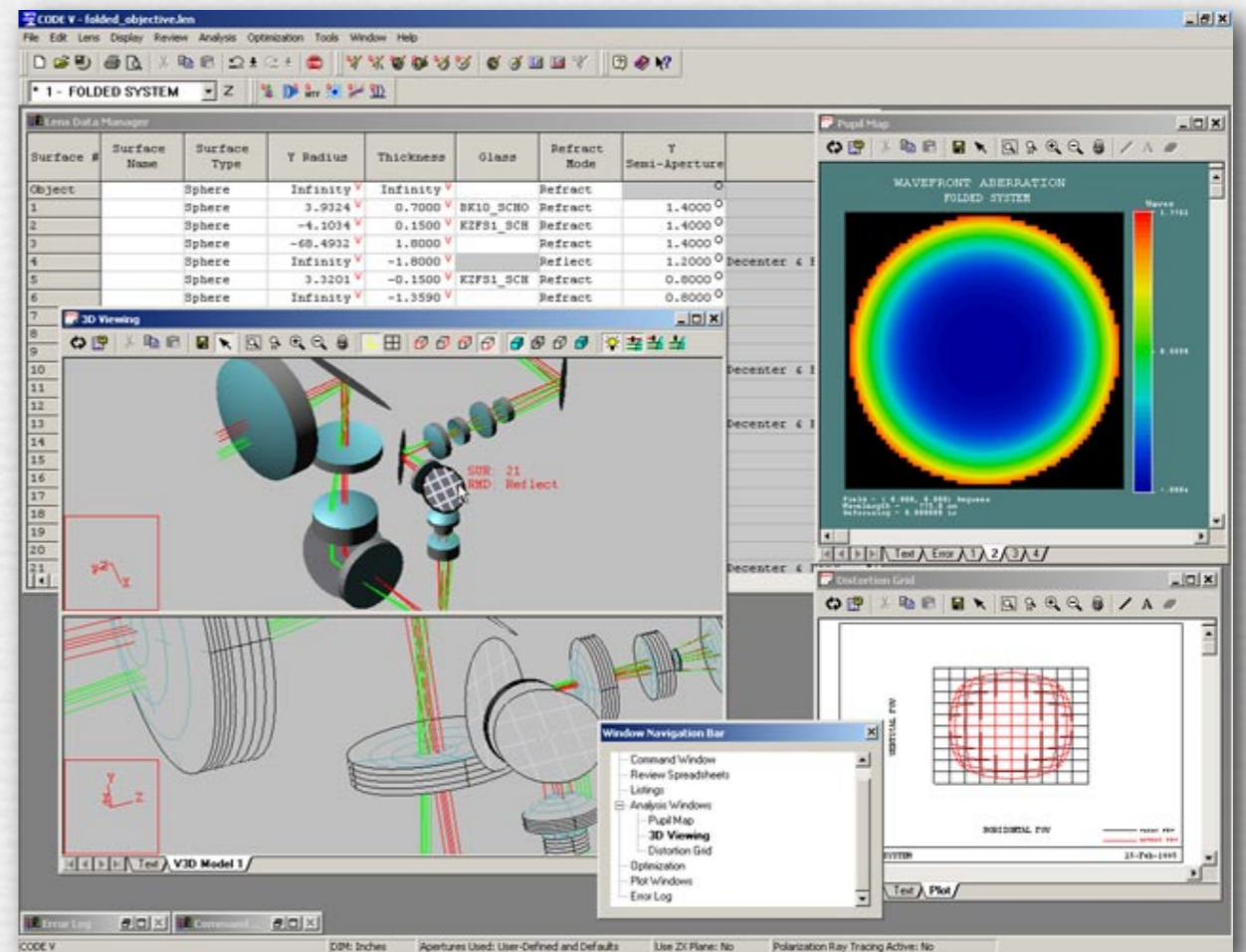
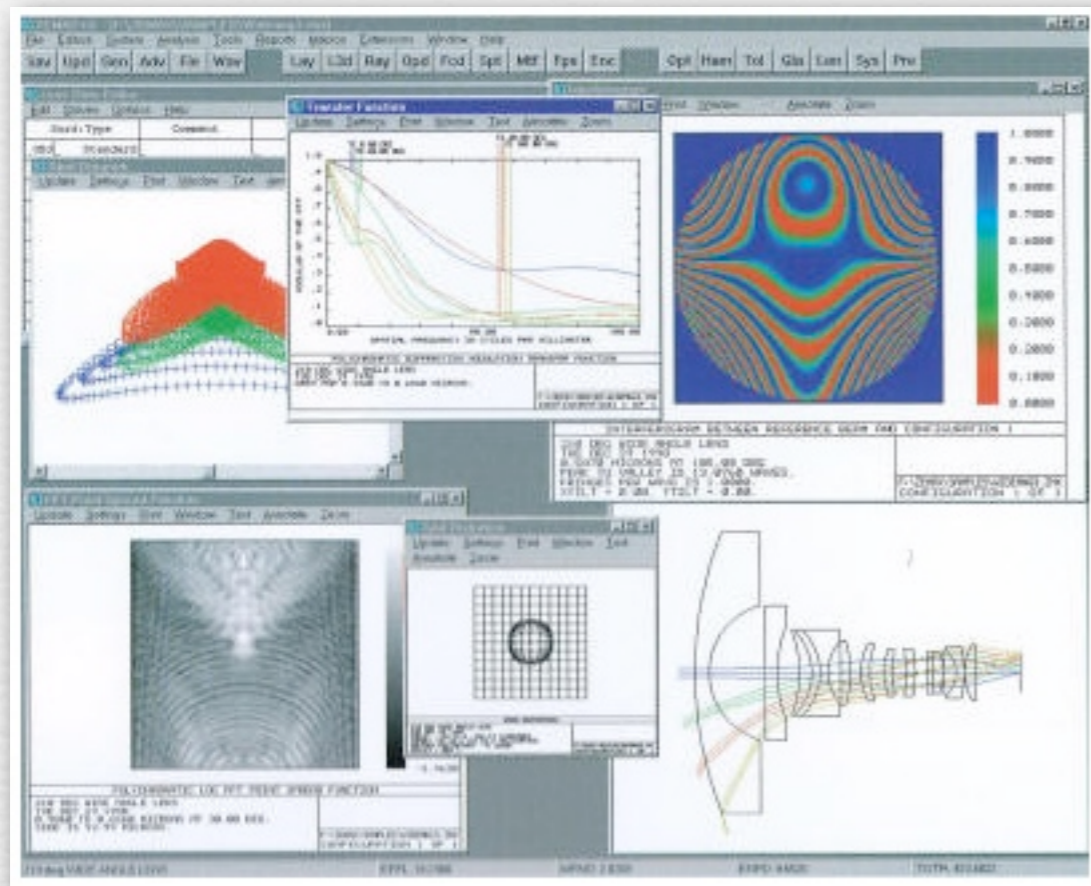
Recap

- ◆ all lenses are subject to chromatic aberration
 - longitudinal appears everywhere; lateral is worse at edges
 - only longitudinal can be reduced by closing down aperture
 - both can be partly corrected using more lenses, and lateral can be partly corrected using software

- ◆ all spherical lenses are subject to Seidel aberrations: spherical, coma, astigmatism, field curvature, distortion
 - some appear everywhere; others only at edges
 - all but distortion can be reduced by closing down aperture
 - only distortion can be corrected completely in software

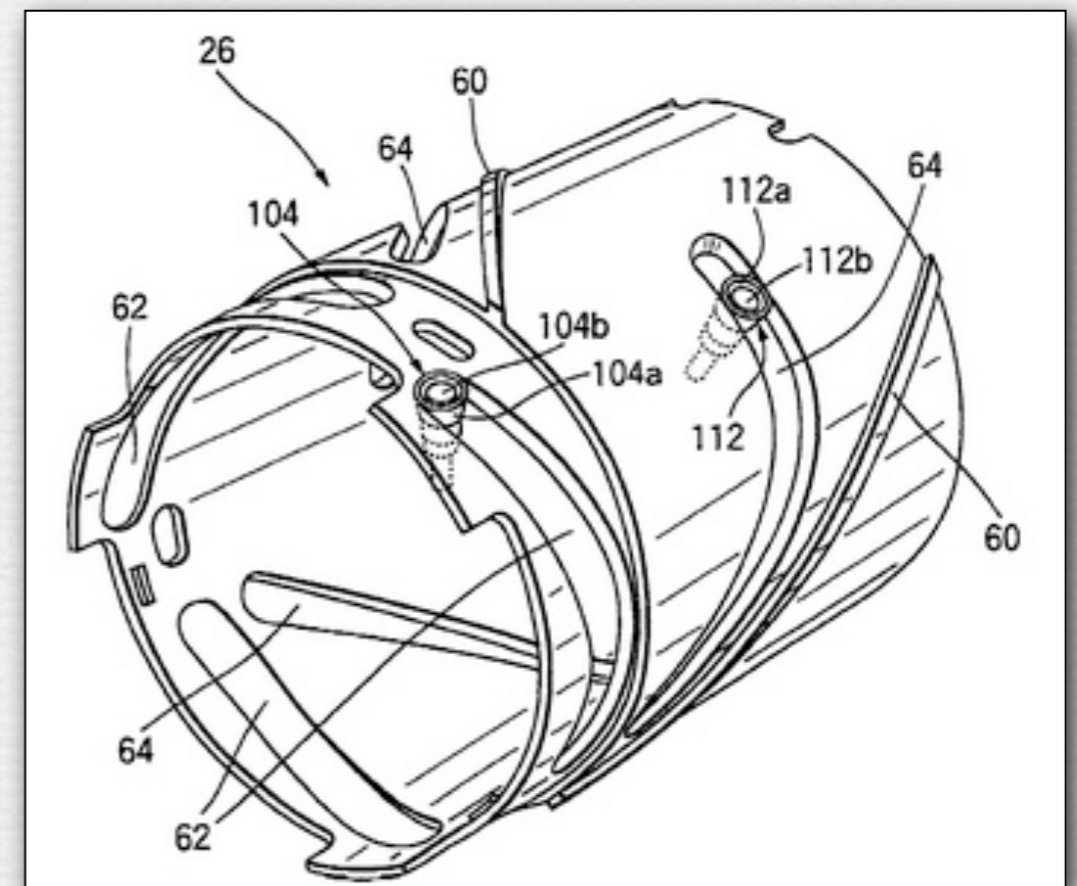
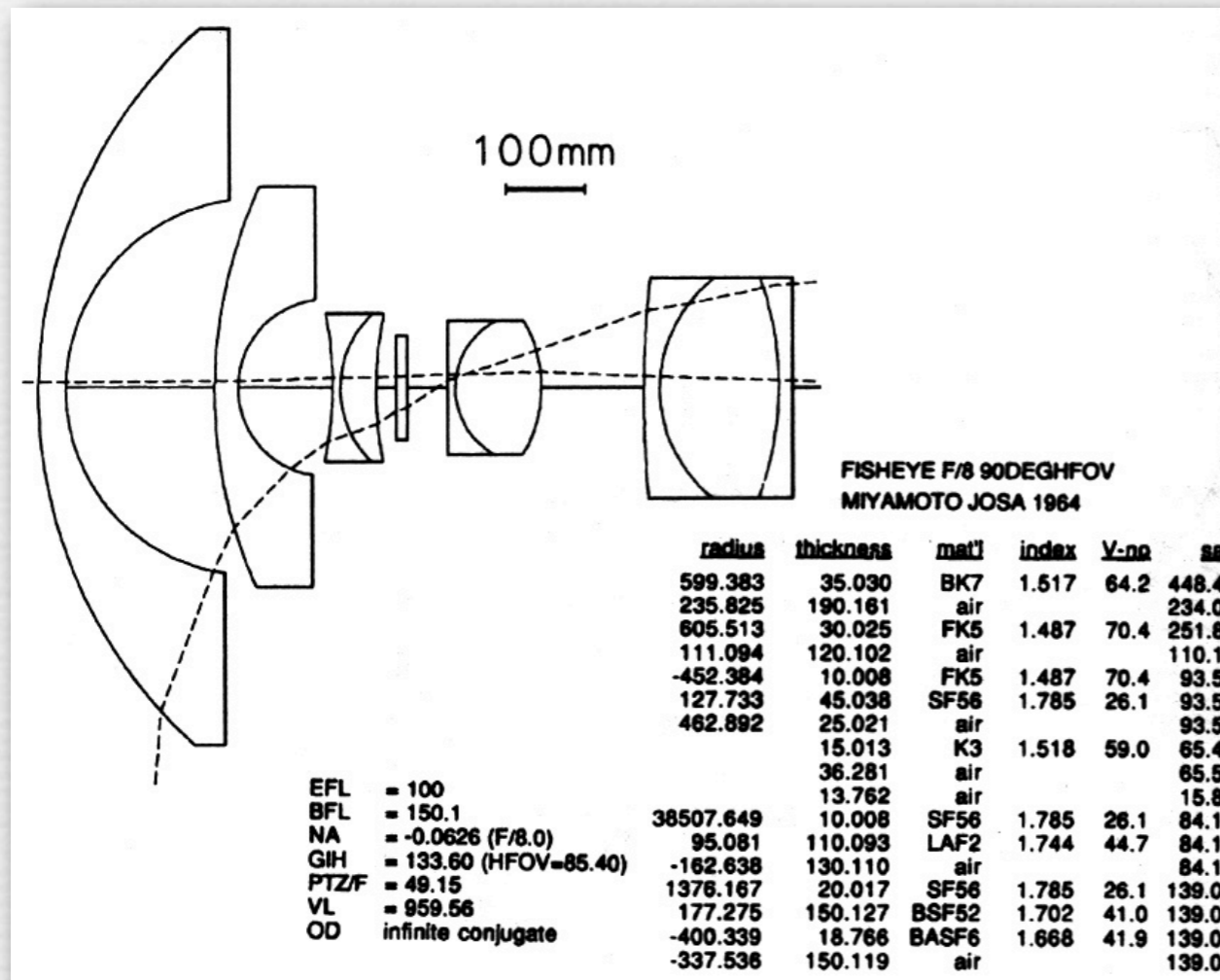
Questions?

Lens design software



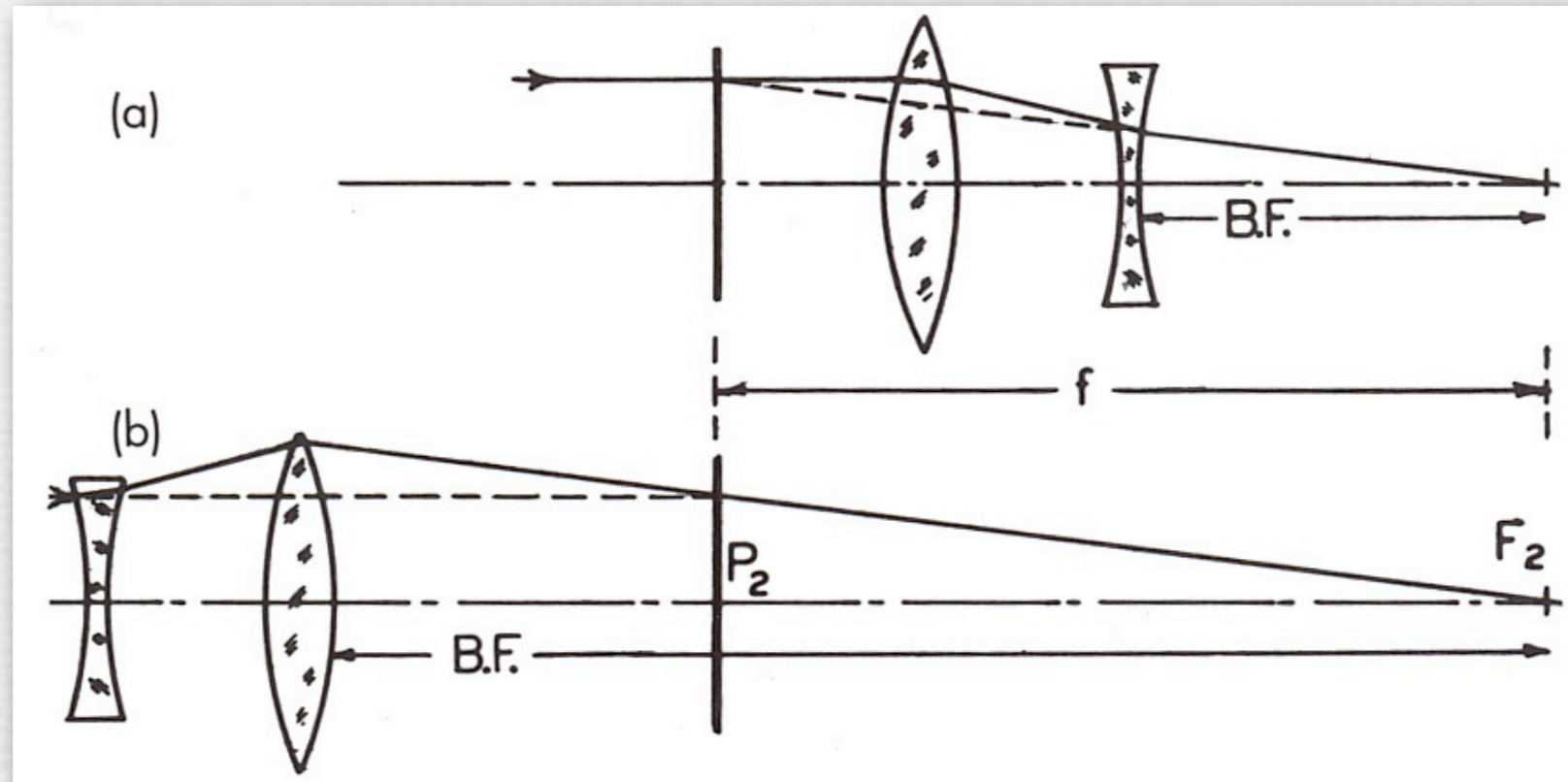
◆ uses optimization to make good recipes better

Lens catalogs and patents



◆ hard to find optical recipe for commercial camera lenses

Lens combinations: telephoto

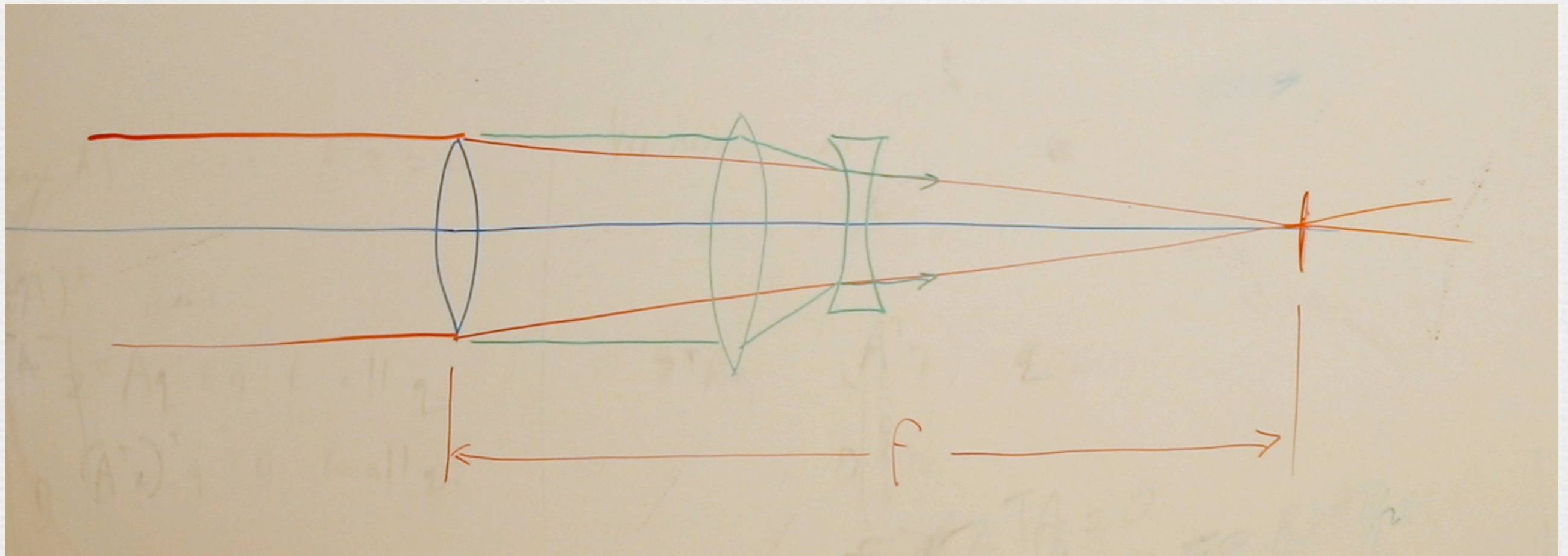


(Kingslake)

- ◆ telephoto (a) reduces the back focal distance B.F. relative to f
 - for long focal length lenses, to reduce their physical size
- ◆ reversed telephoto (b) increases B.F. relative to f
 - for wide-angle lenses, to ensure room for the reflex mirror

Telephoto lens

- ◆ the blue lens is replaced with the two green ones, thereby reducing the physical size of the lens assembly, while preserving its focal length (hence magnification)



Lens combinations: telephoto

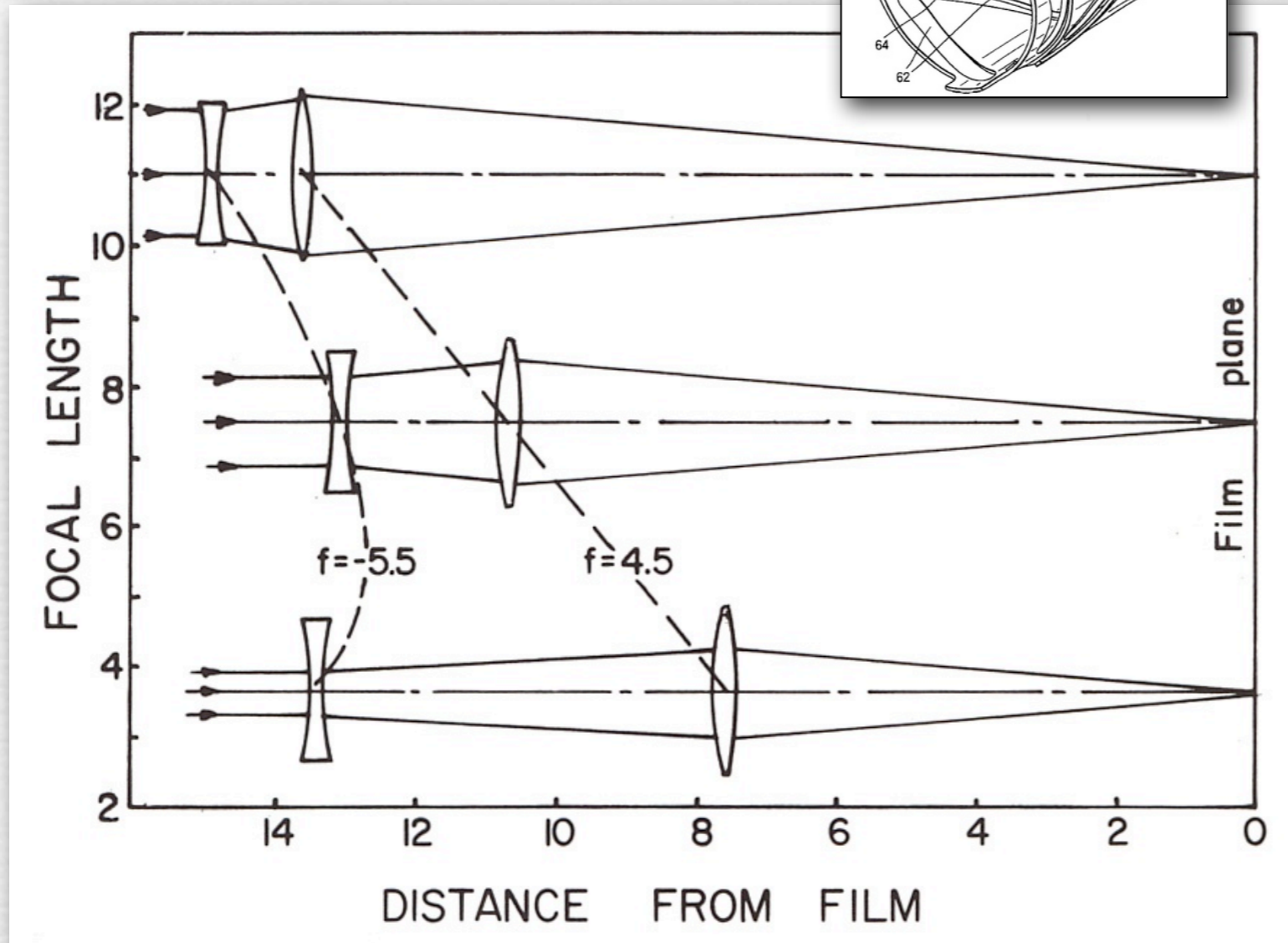
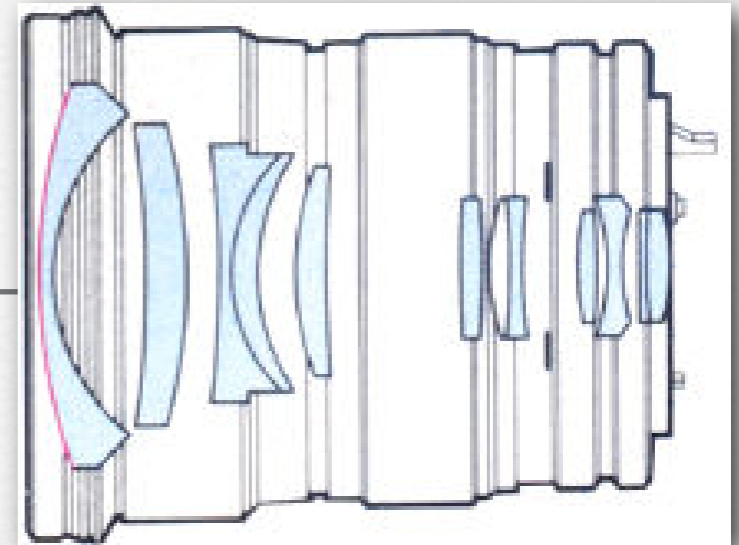
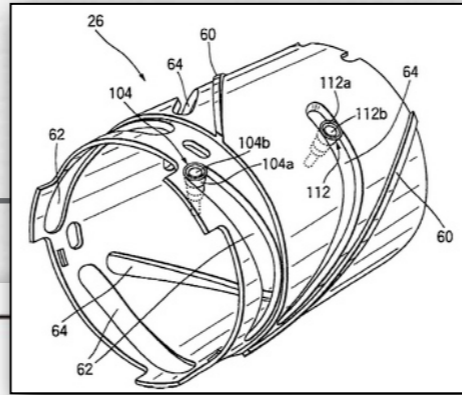


Nikon 500mm telephoto

Opteka 500mm non-telephoto



Lens combinations: zoom



Canon FD 24-35mm
f/3.5 L manual focus lens

(FLASH DEMO)

<http://graphics.stanford.edu/courses/cs178/applets/zoom.html>

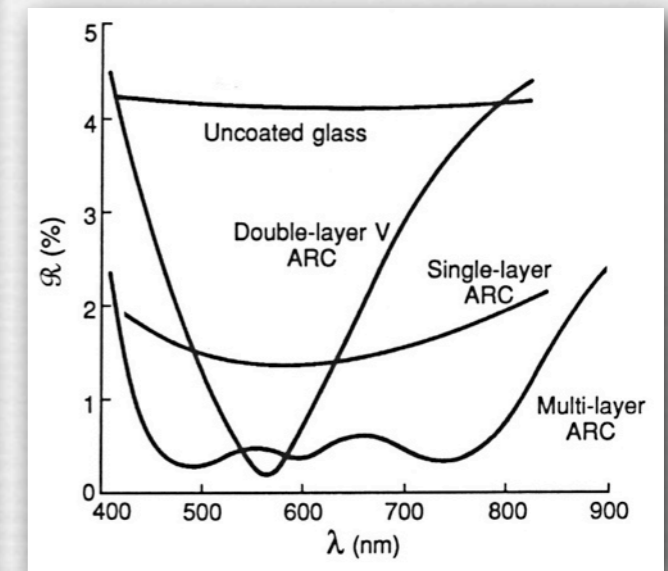
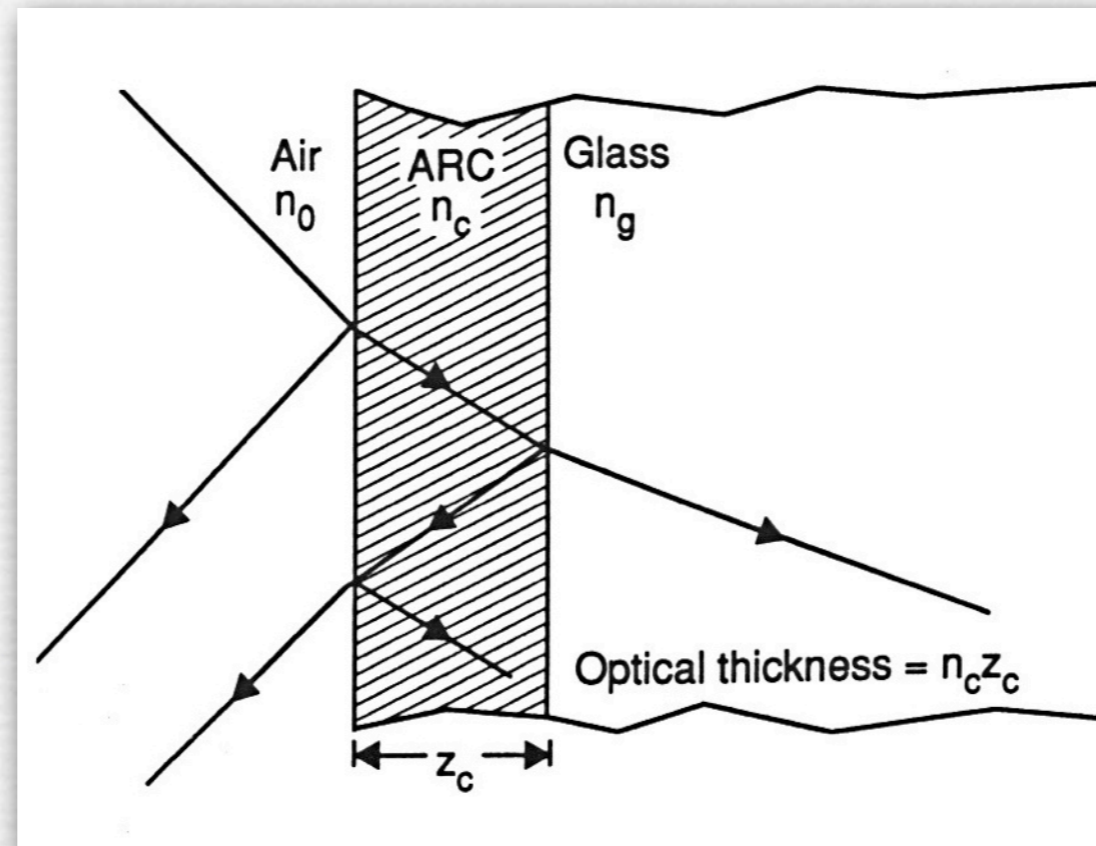
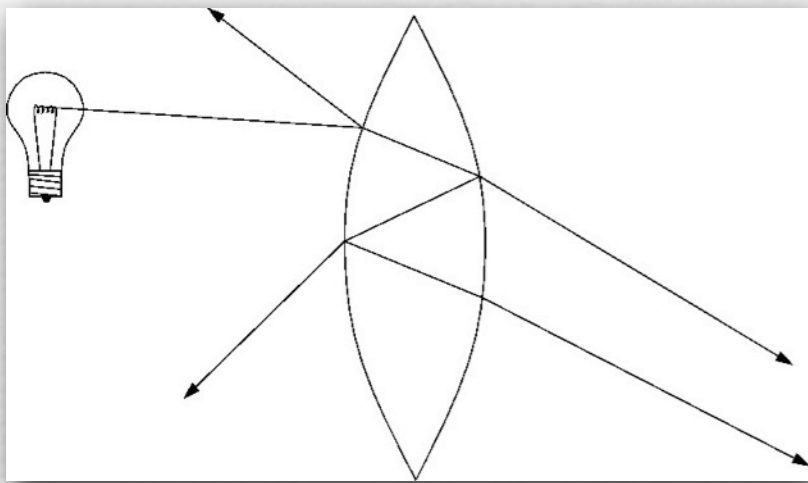
- ◆ called *optically compensated zoom*, because the in-focus plane stays (more or less) stationary as you zoom
- ◆ to change focus, you move both lenses together

Recap

- ◆ telephoto lenses separate focal length & back focal distance
 - for long focal length lenses, to reduce their physical size
 - for wide-angle lenses, to ensure room for the reflex mirror

Questions?

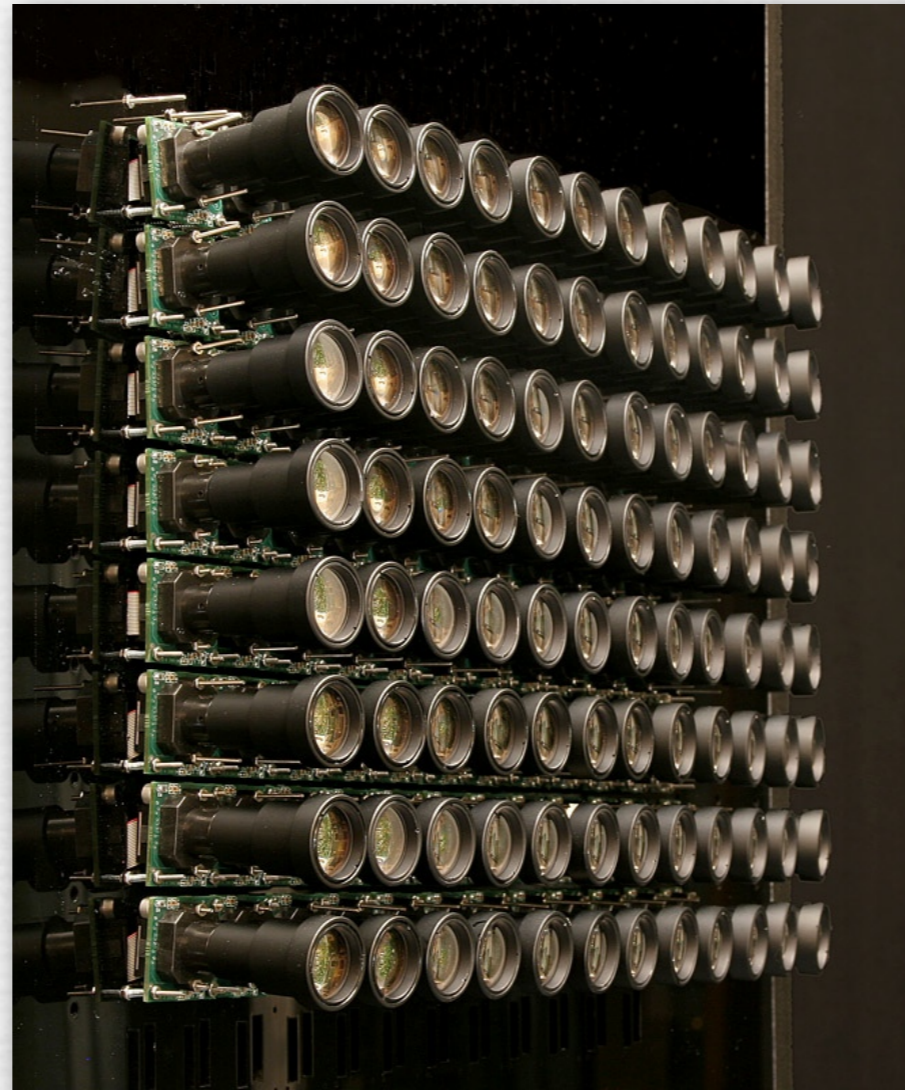
Veiling glare



- ◆ contrast reduction caused by stray reflections
- ◆ can be reduced by anti-reflection coatings
 - based on interference, so optimized for one wavelength
 - to cover more wavelengths, use multiple coatings

Camera array with too much glare

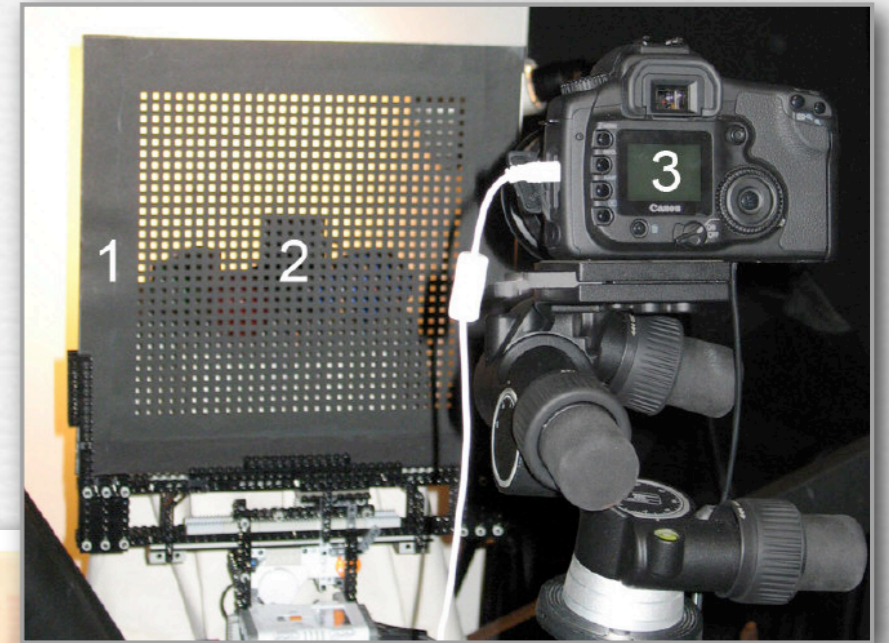
Stanford Multi-Camera Array



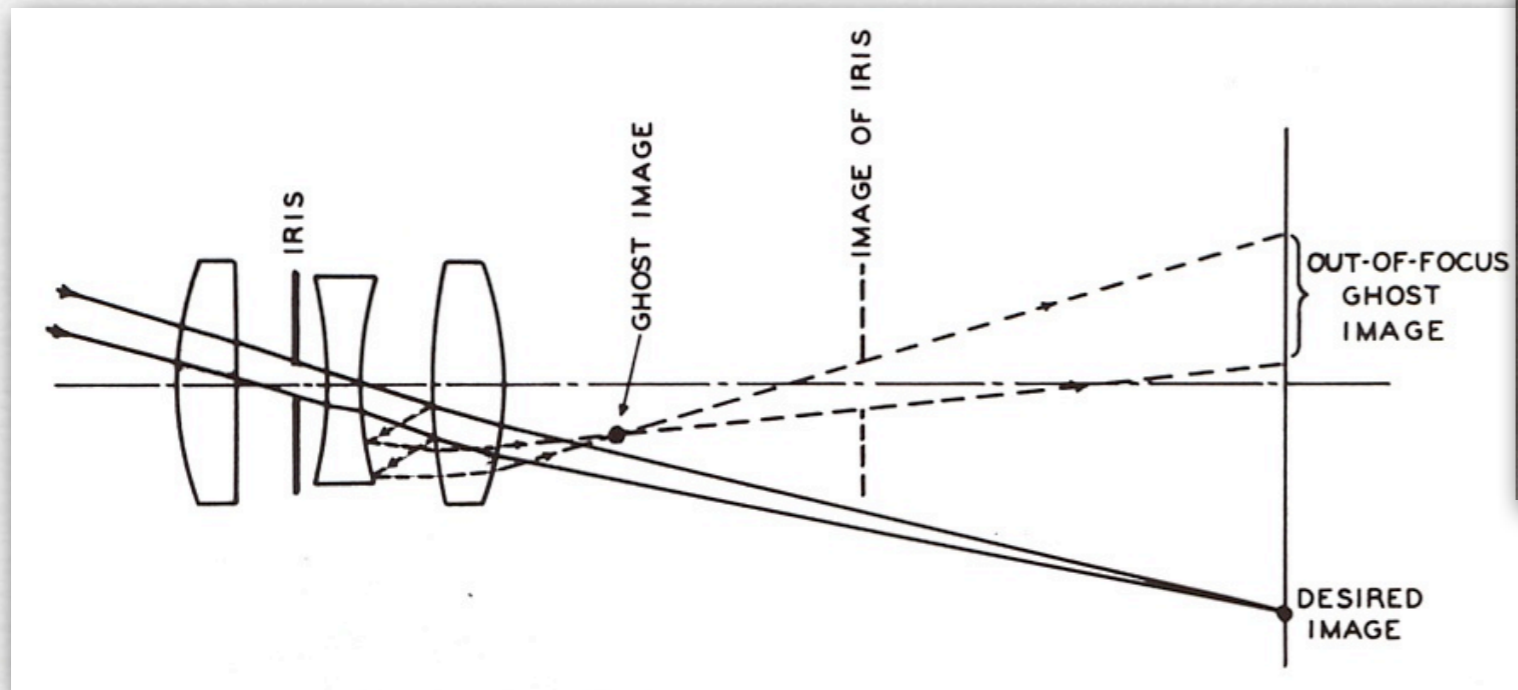
- ◆ 12×8 array of 600×800 pixel webcams = $7,200 \times 6,400$ pixels
- ◆ goal was highest-resolution movie camera in the world
- ◆ failed because glare in inexpensive lenses led to poor contrast

Removing veiling glare computationally

[Talvala, Proc. SIGGRAPH 2007]



Flare and ghost images

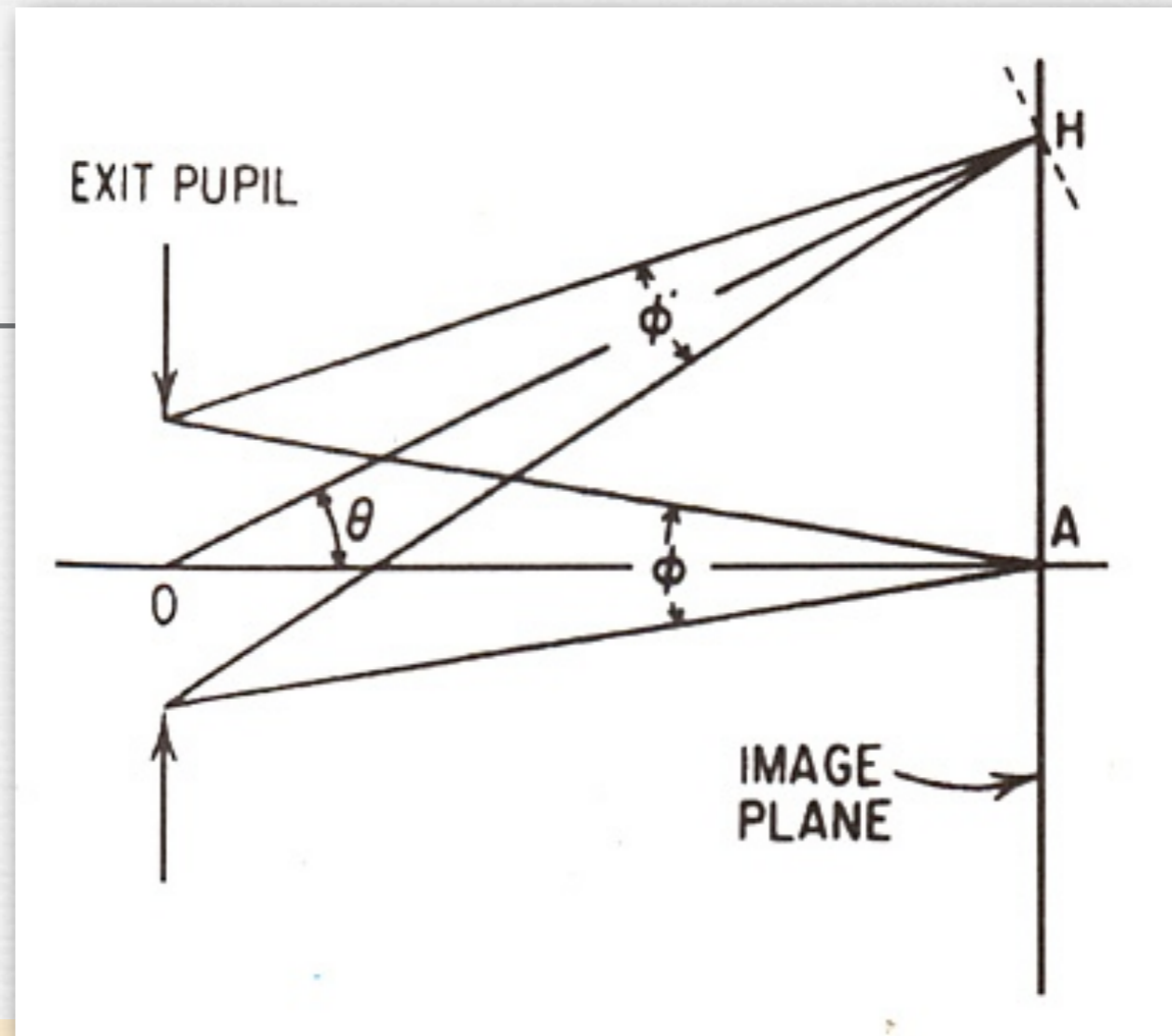


(Kingslake)

- ◆ reflections of the aperture, lens boundaries, etc., i.e. things inside the camera body
- ◆ removing these artifacts is an active area of research in computational photography
- ◆ but it's a hard problem

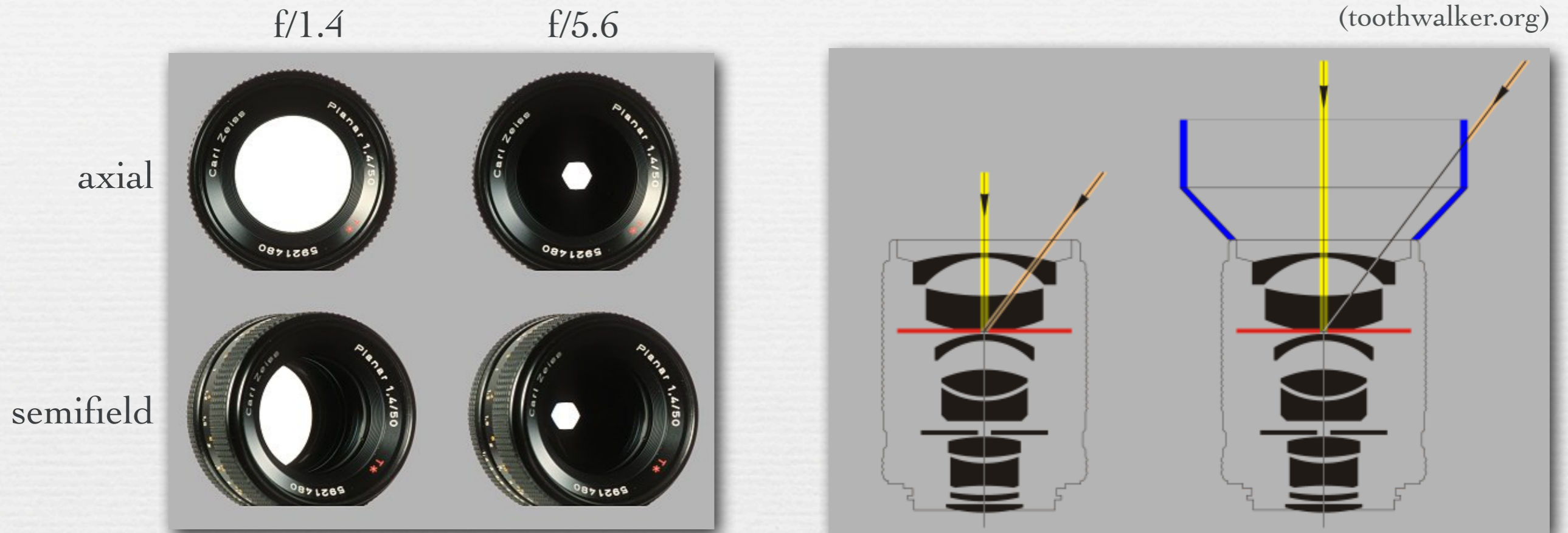
Vignetting

(a.k.a. natural vignetting)



- ◆ irradiance is proportional to projected area of aperture as seen from pixel on sensor, which drops as $\cos \theta$
- ◆ irradiance is proportional to projected area of pixel as seen from aperture, which also drops as $\cos \theta$
- ◆ irradiance is proportional to distance² from aperture to pixel, which rises as $1/\cos \theta$
- ◆ combining all these effects, light drops as $\cos^4 \theta$

Other sources of vignetting



from multiple lens elements,
especially at wide apertures

from add-on lens hoods
(or filters or fingers)

- ◆ pixel vignetting due to shadowing inside each pixel
(we'll come back to this)

Examples



(toothwalker.org)



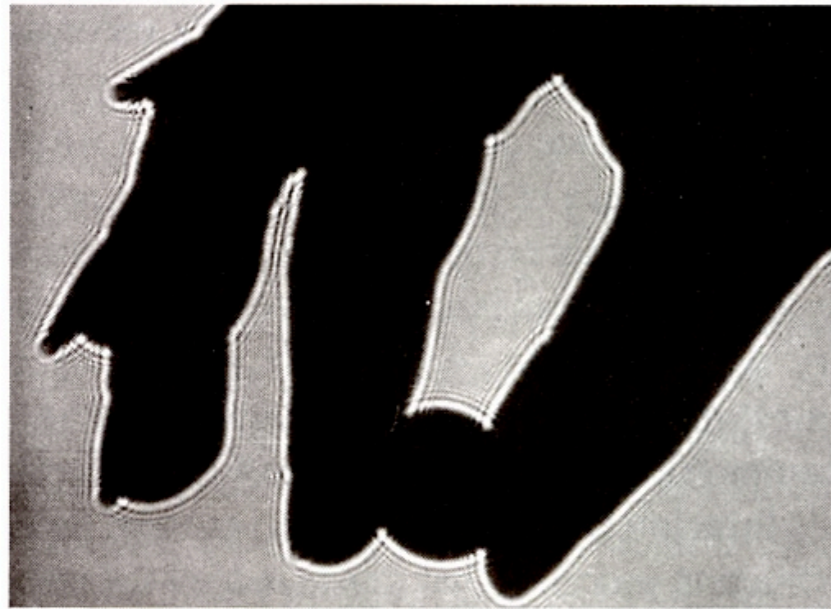
(toothwalker.org)



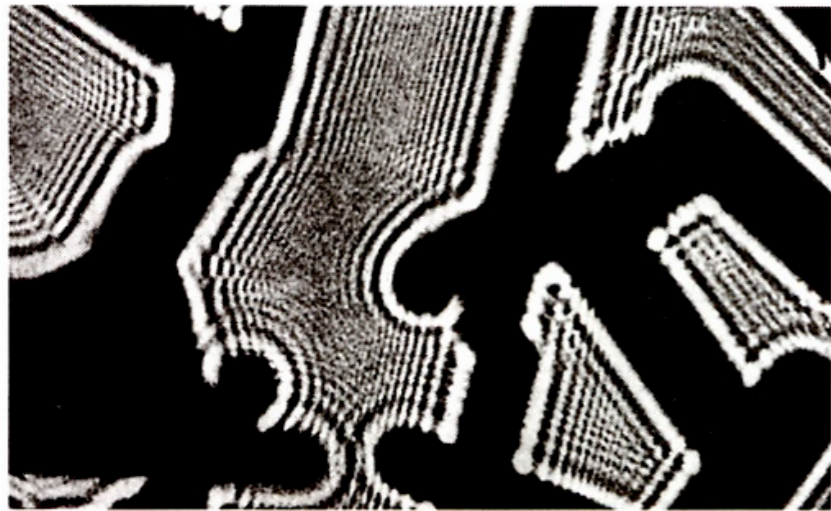
(wikipedia)

- ◆ vignetting affects the *bokeh* of out-of-focus features
- ◆ vignetting is correctable in software (except for bokeh effects), but boosting pixel values worsens noise
- ◆ vignetting can be applied afterwards, for artistic purposes

Diffraction

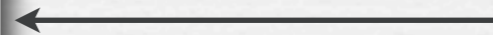


(a)

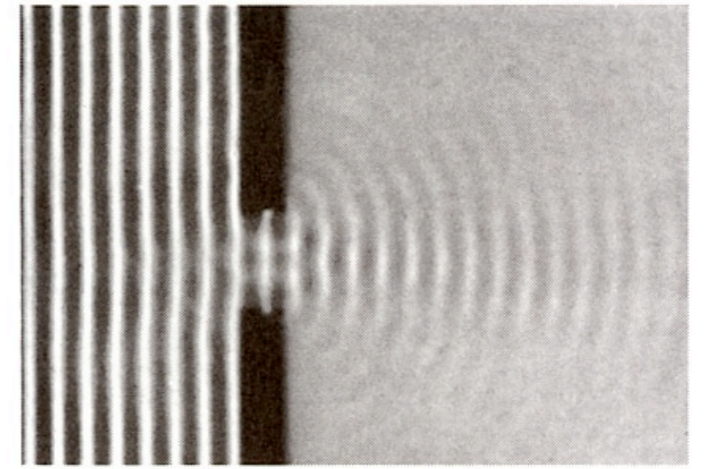


(b)

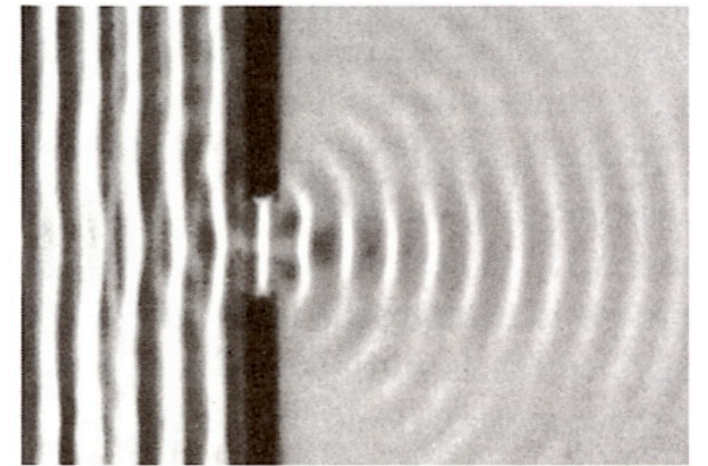
illuminated by a
(spread-out) laser beam
& recorded directly on film



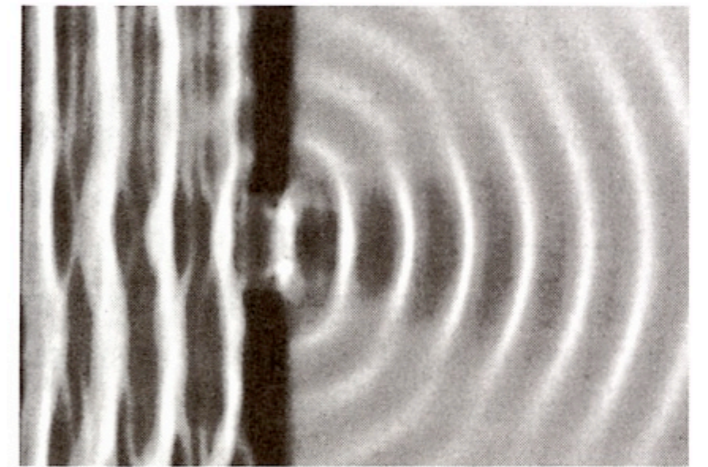
varying the wavelength
of waves passing through
a slit in a ripple tank



(a)



(b)

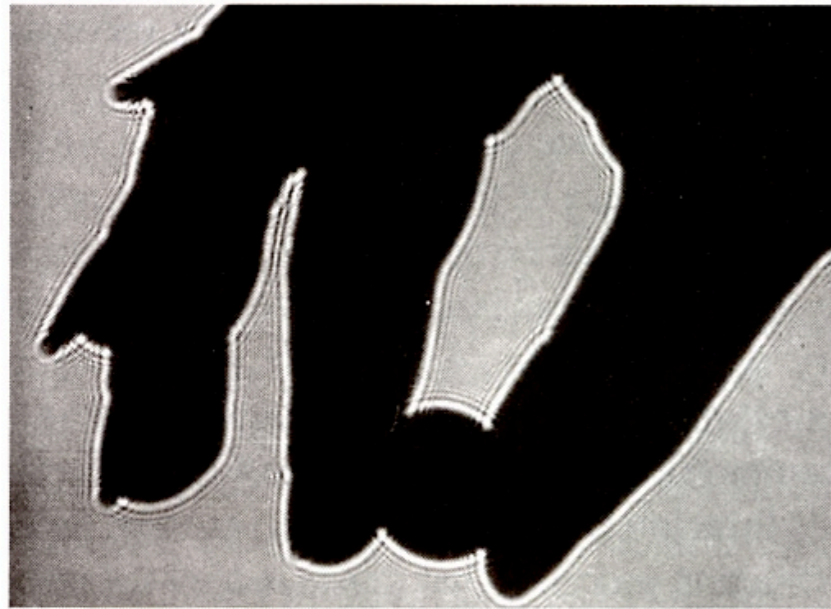


(c)

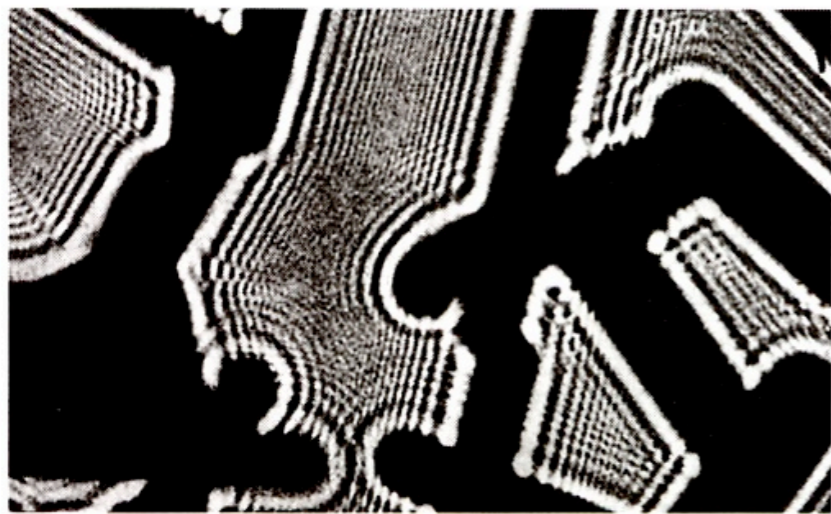
(Hecht)

- ◆ as wavelength decreases in the ripple tank, propagation becomes more ray-like

Diffraction



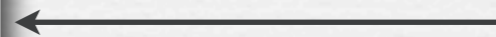
(a)



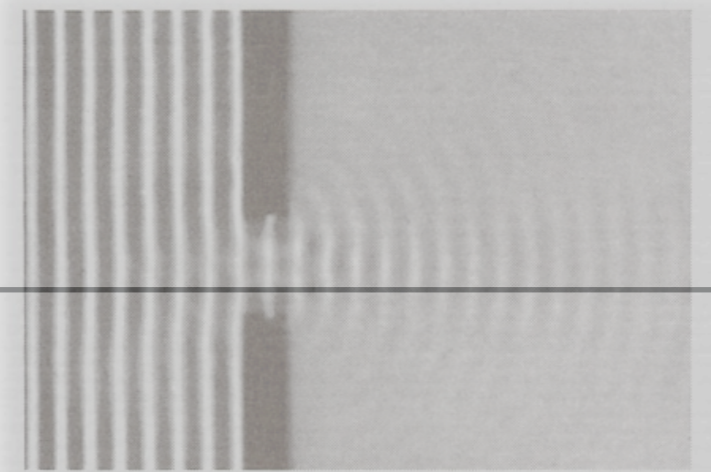
(b)

(Hecht)

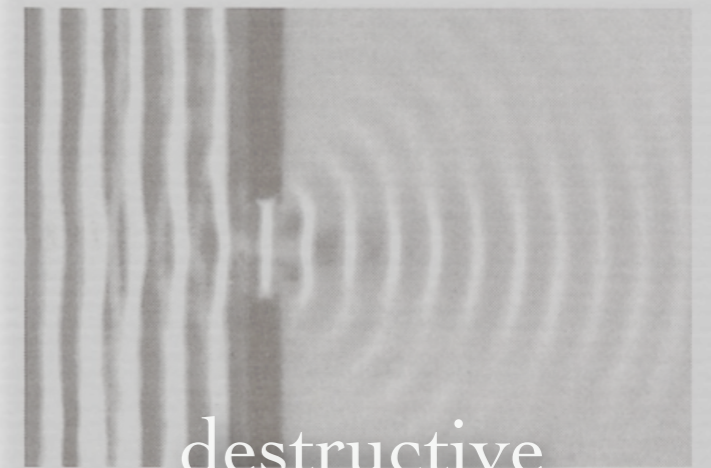
illuminated by a
(spread-out) laser beam
& recorded directly on film



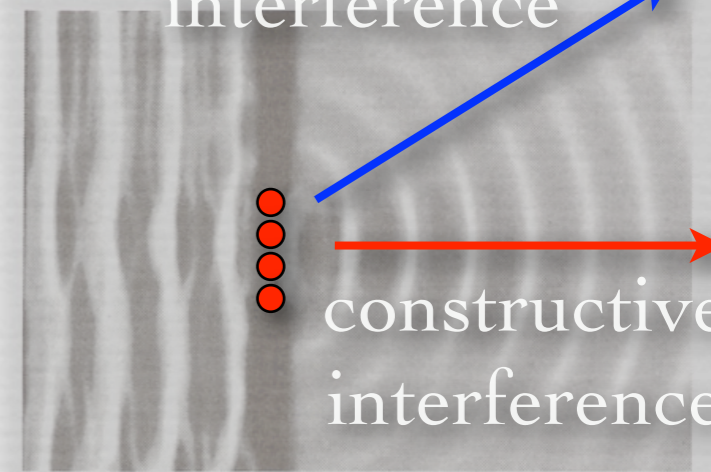
varying the wavelength
of waves passing through
a slit in a ripple tank



(a)



destructive
interference^(b)

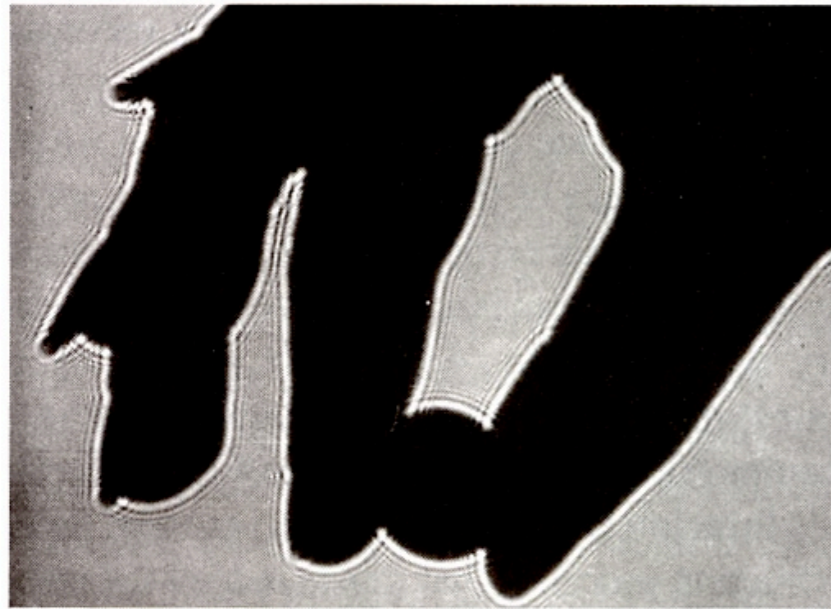


constructive
interference

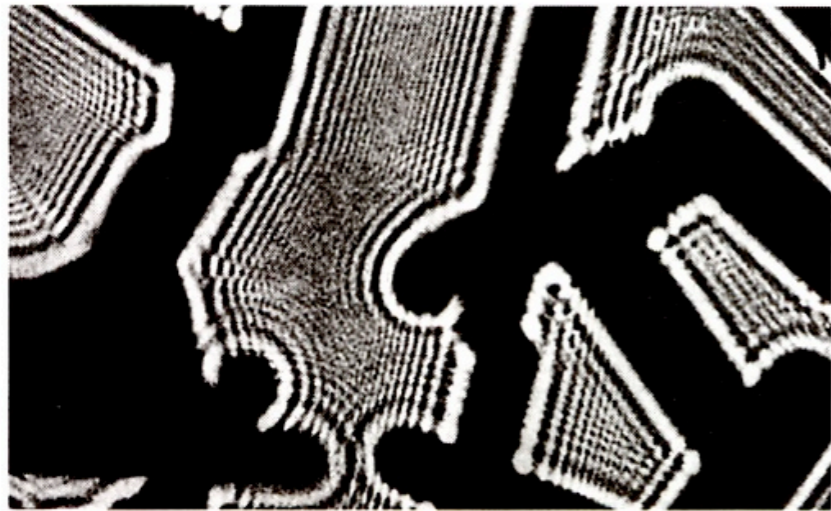
(c)

- ◆ as wavelength decreases in the ripple tank, propagation becomes more ray-like

Diffraction



(a)



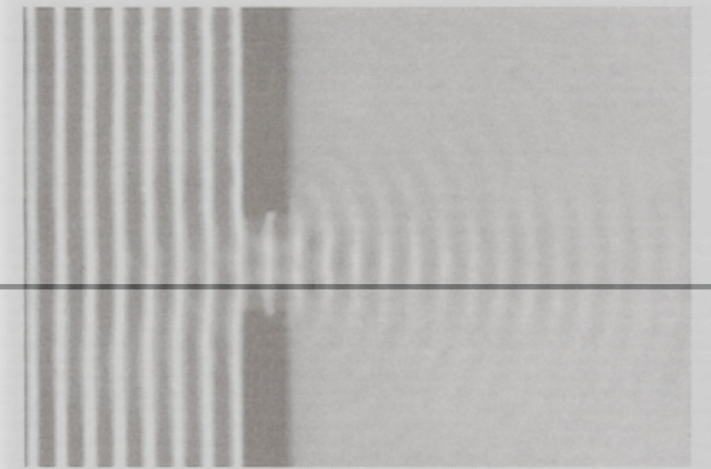
(b)

(Hecht)

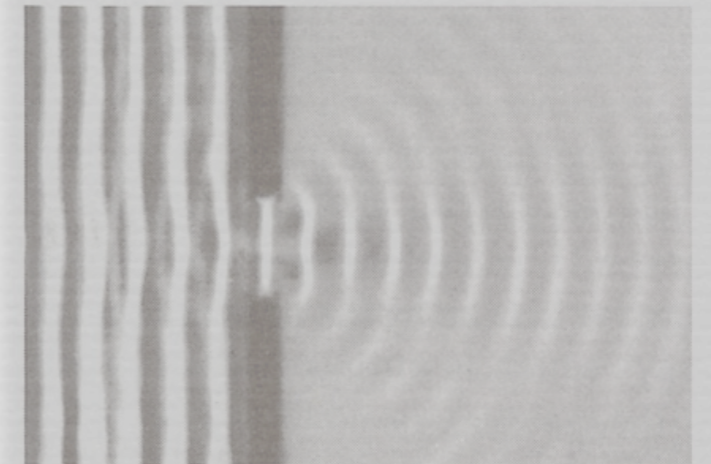
illuminated by a
(spread-out) laser beam
& recorded directly on film



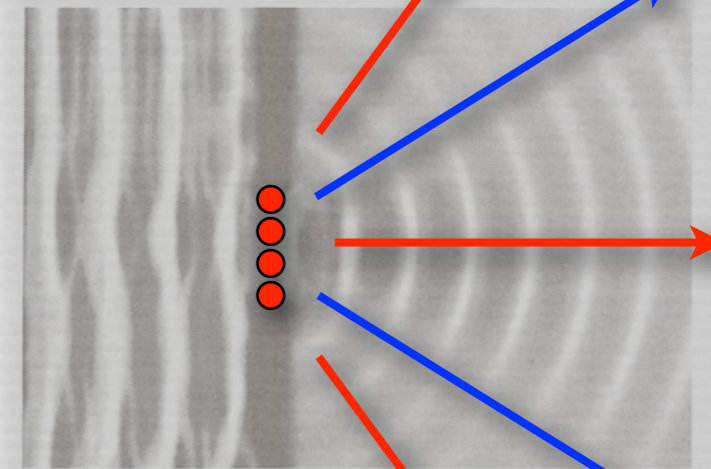
varying the wavelength
of waves passing through
a slit in a ripple tank



(a)



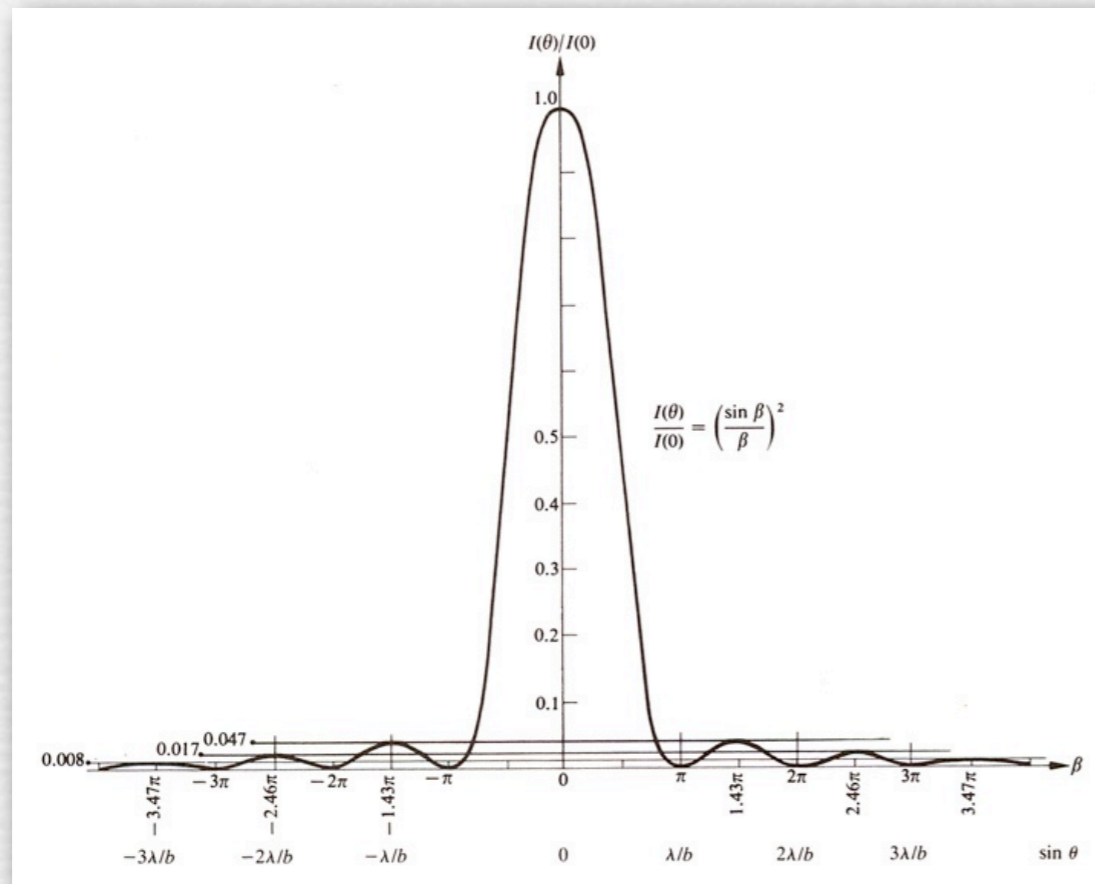
(b)



(c)

- ◆ as wavelength decreases in the ripple tank, propagation becomes more ray-like

Airy rings



diffraction from a slit



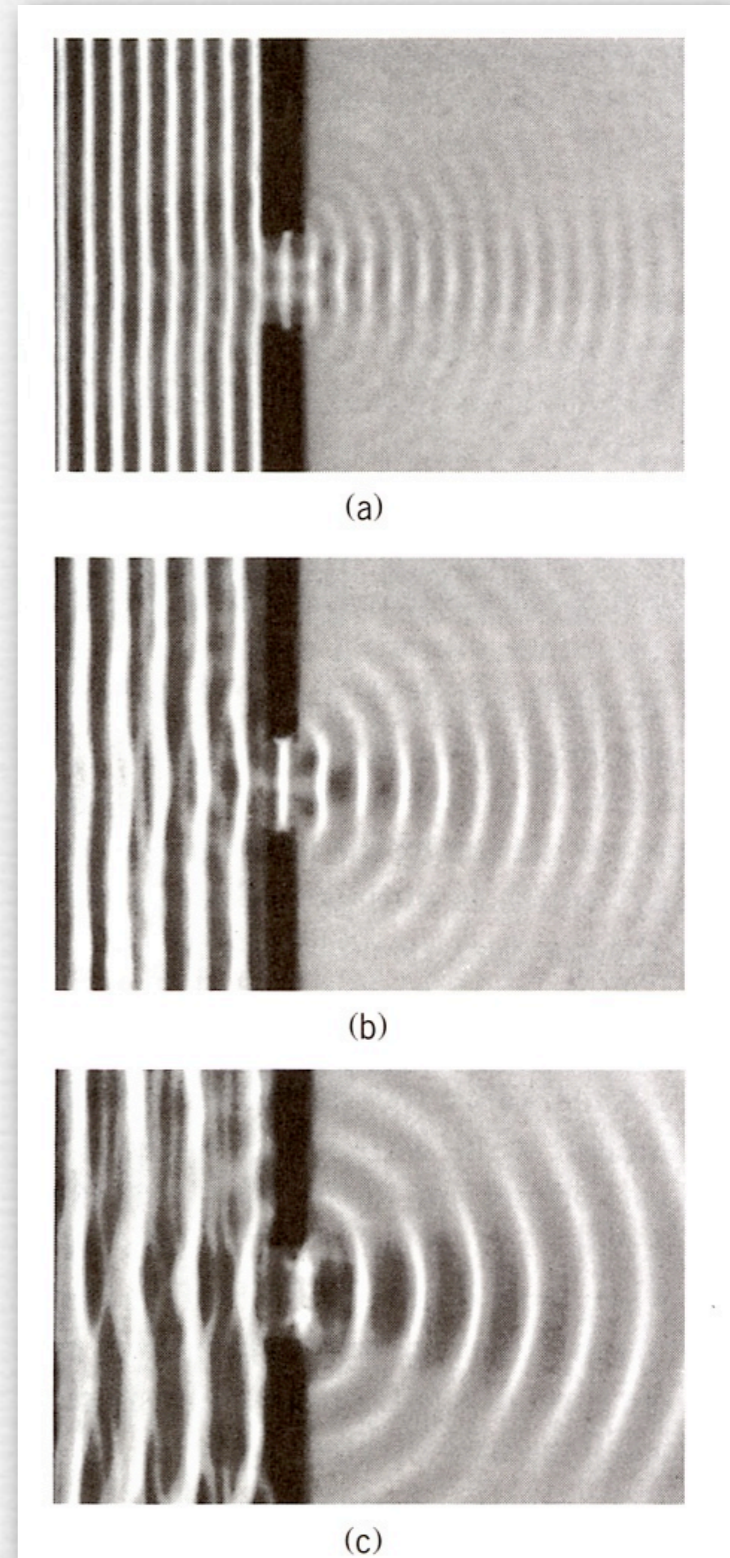
(Hecht)

diffraction from a circular aperture: Airy rings

- ◆ if the illumination were a laser, a lens would produce this pattern
- ◆ but considering all wavelengths, the dark rings vanish, leaving a blur

Diffraction in photographic cameras

- ◆ well-corrected lenses are called *diffraction-limited*
- ◆ the smaller the aperture (A) (or the longer the wavelength), the larger the diffraction blur
- ◆ the longer the distance to the sensor (f), the larger the blur
- ◆ thus, the size of the blur varies with $N = f / A$



Examples



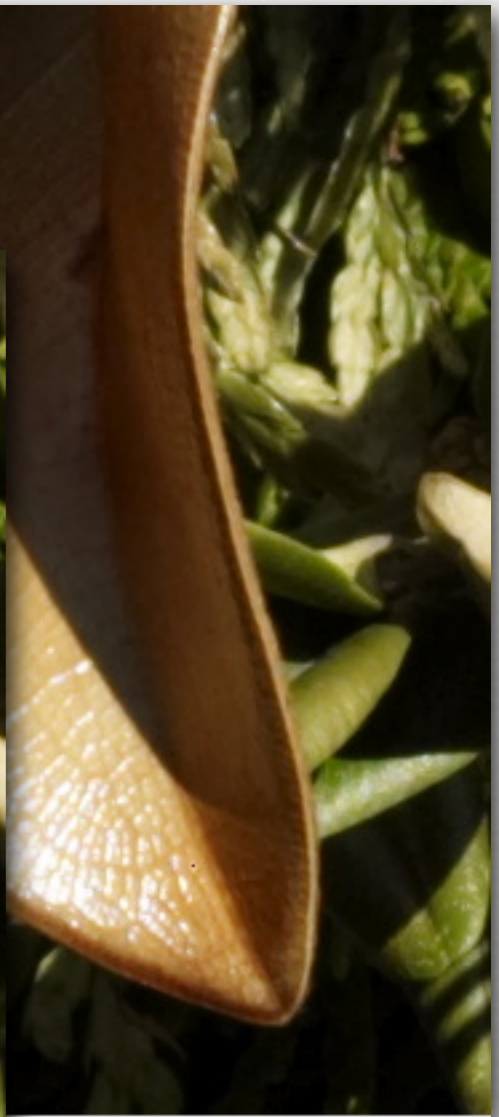
f/22



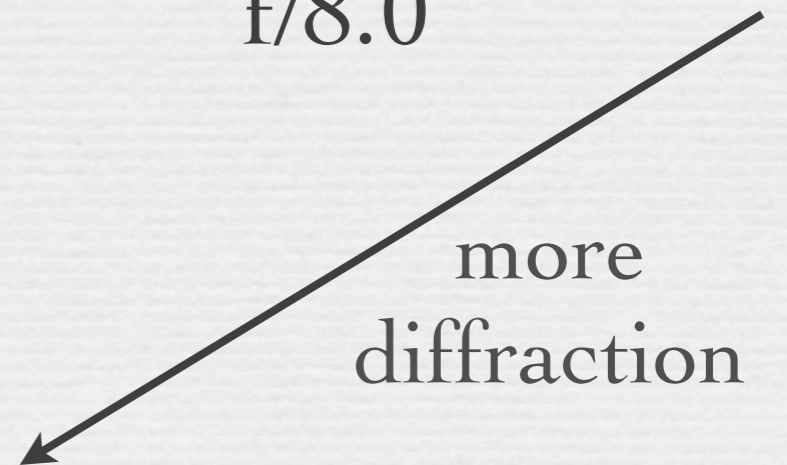
f/11



f/8.0



f/5.6



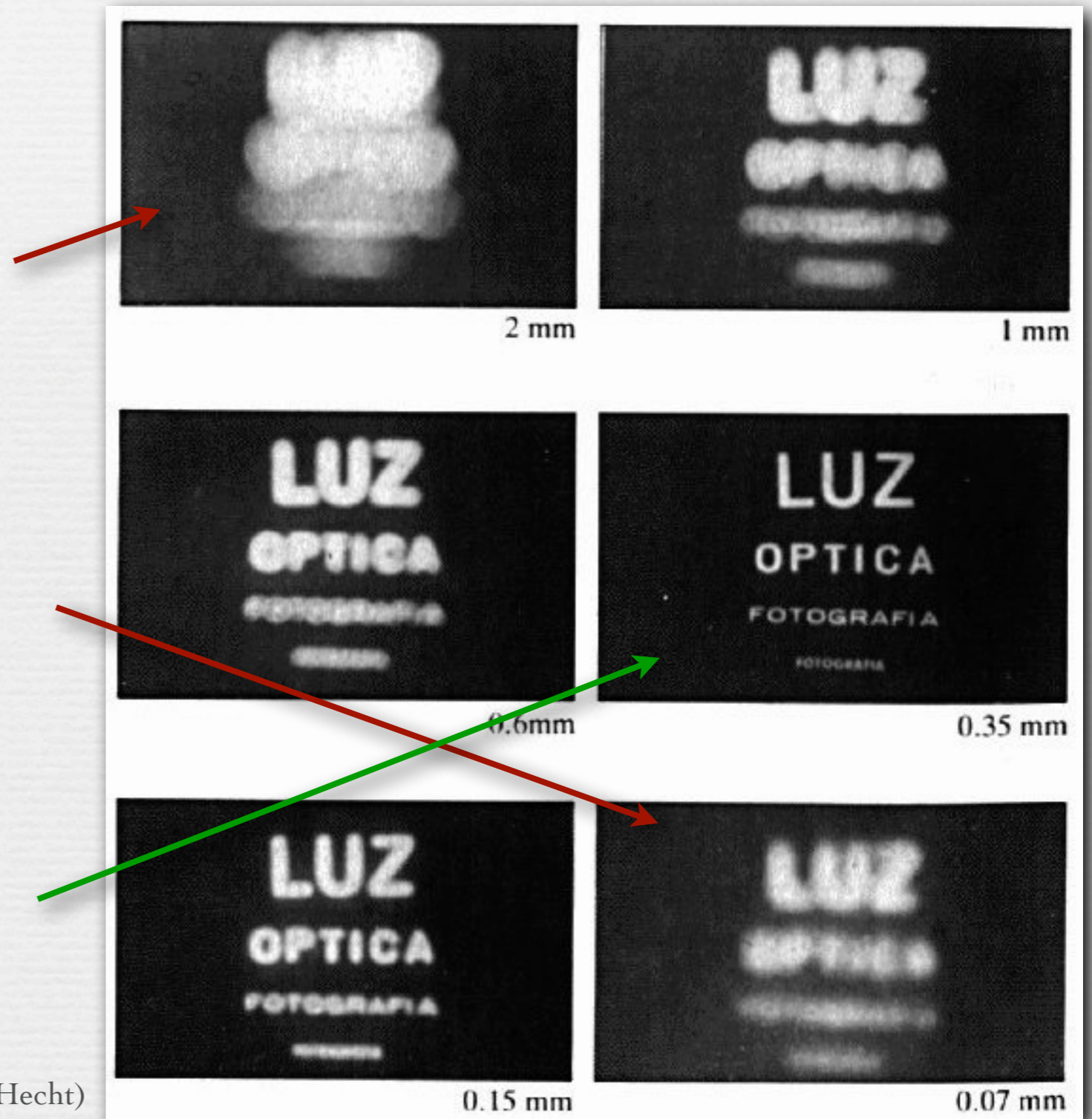
(luminous-landscape.com)

Effect of pinhole size

◆ large pinhole
gives geometric blur

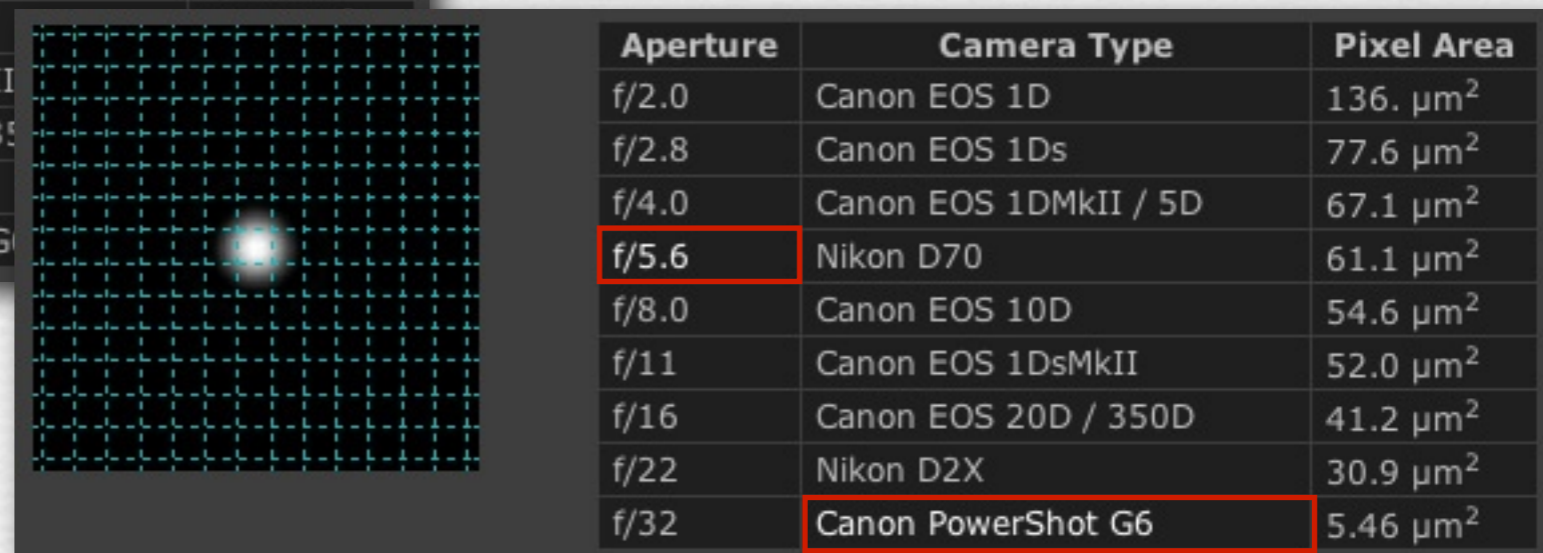
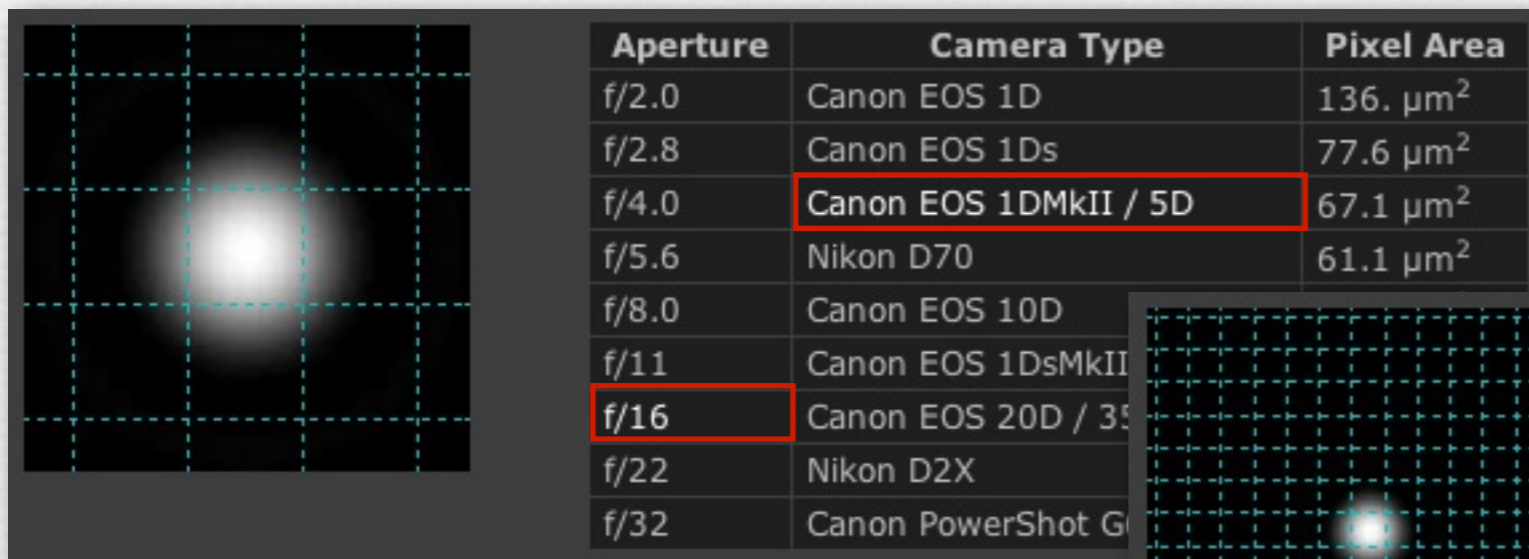
◆ small pinhole
gives diffraction blur

◆ optimal pinhole
gives very little light



Diffraction in photographic cameras

- ◆ the smaller the pixels, the more of them the pattern covers
 - if the pattern spans $\gg 1$ pixel, we begin to complain



(<http://www.cambridgeincolour.com/tutorials/diffraction-photography.htm>)

The Abbe diffraction limit

$$d = \frac{.61 \lambda}{NA} \approx 1.2 N \lambda$$

◆ where

- λ = wavelength
- NA = numerical aperture $\approx 1 / 2N$

◆ Example: iPhone 4 when looking at green

- $\lambda = 550\text{nm}$
- $N = f/3$
- $d = 2\mu$
- pixels are 1.75μ wide, so the iPhone 4 would be roughly diffraction-limited if its lenses were free of aberrations

Recap

- ◆ all optical systems suffer from veiling glare
 - anti-reflection coatings help
- ◆ all optical systems suffer from flare and ghosts
 - don't point your camera at bright lights; use lens hoods
- ◆ vignetting arises from many sources
 - natural - falloff at the edges of wide sensors
 - optical - caused by apertures, lens barrels
 - mechanical - caused by wrong lens hoods, hands, straps
 - pixel - caused by shadowing inside pixel structures
- ◆ diffraction - blur that varies with $N = f / A$
 - avoid F-numbers above f/16 (for full-frame camera)
 - subjective image quality depends on both sharpness and contrast

Questions?

Slide credits

◆ Marc Levoy

◆ Steve Marschner

◆ Fredo Durand

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◆ Kemp, M., *The Science of Art*, Yale University Press, 1990.

◆ Hecht, E., *Optics* (4th ed.), Pearson / Addison-Wesley, 2002.

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◆ Pamplona et al., "NETRA: Interactive Display for Estimating Refractive Errors and Focal Range", *Proc. SIGGRAPH 2010*.

◆ <http://dpreview.com>